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Report of Meeting

The Tenth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities, Zamárdi (Hungary), February 3–6, 2010

The Tenth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities was held in Művész Üdülő, Zamárdi, Hungary, from February 3 to 6, 2010. It was organized by the Department of Analysis of the Institute of Mathematics of the University of Debrecen.

24 participants came from the University of Debrecen (Hungary) and the Silesian University of Katowice (Poland) at 12 from each of both cities.

Professor Zsolt Páles opened the Seminar and welcomed the participants to Zamárdi.

The scientific talks presented at the Seminar focused on the following topics: equations in a single variable and in several variables, iterative equations, equations on algebraic structures, regularity properties of the solutions of certain functional equations, functional inequalities, Hyers–Ulam stability, functional equations and inequalities involving mean values, generalized convexity. Interesting discussions were generated by the talks.

There was a profitable Problem Session.

The social program included a banquet and an excursion which consisted of visiting the thermal lake in Hévíz. The closing address was given by Professor Roman Ger. His invitation to the Eleventh Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities in February 2011 in Poland was gratefully accepted.

Summaries of the talks in alphabetic order of the authors follow in Section 1, problems and remarks in chronological order in Section 2, and the list of participants in the final section.

1. Abstracts of talks

Roman Badora: *Stability and separation theorems*

In this talk we discuss the problem of connections between stability results and separation theorems.

Szabolcs Baják: *Invariance equations involving Gini and Stolarsky means* (Joint work with Zsolt Páles)

Given three strict means $M, N, K\colon \mathbb{R}_+^2 \to \mathbb{R}_+$, we say that the triple (M, N, K) satisfies the *invariance equation* if

$$K\big(M(x,y), N(x,y)\big) = K(x,y), \quad x, y \in \mathbb{R}_+,$$

holds. It is well known that K is uniquely determined by M and N, and it is called the Gauss composition $K = M \otimes N$ of M and N.

Our aim is to solve the invariance equation when each of the means M, N, K is either a Gini or a Stolarsky mean with possibly different parameters. This means that we have to consider six different equations. With the help of the computer algebra system Maple V Release 9, we give the general solutions of these equations.

Zoltán Boros: *Weakly affine functions*

We call a function $f\colon \mathbb{R} \to \mathbb{R}$ *weakly affine* if, for every $x, y \in \mathbb{R}$, there exists a real number t (depending on x and y) such that $0 < t < 1$ and

$$f\left(tx + (1-t)y\right) = tf(x) + (1-t)f(y). \tag{1}$$

We call f *affine* if (1) holds for every $t \in [0,1]$ and $x, y \in \mathbb{R}$. If (1) is satisfied by $t = 1/2$ and every $x, y \in \mathbb{R}$, we say that f is a *midpoint-affine* function.

Clearly, every midpoint-affine function is also weakly affine. It is well known ([1, Theorem 89, sec. 3.7, p. 73]) that every continuous, weakly affine function has to be affine as well. We present an example for a measurable, weakly affine function which is not midpoint-affine. Further regularity assumptions for weakly affine functions are also investigated.

Reference

[1] Hardy G.H., Littlewood J.E., Pólya G., *Inequalities*, University Press, Cambridge, 1934.

Pál Burai: *On approximately h-convex functions* (Joint work with Attila Házy)

A real valued function $f : D \to \mathbb{R}$ defined on an open convex subset D of a normed space X is called *rationally* (h, d)*-convex* if it satisfies

$$f\left(tx + (1-t)y\right) \le h(t)f(x) + h(1-t)f(y) + d(x, y)$$

for all $x, y \in D$ and $t \in \mathbb{Q} \cap [0, 1]$, where $d\colon X \times X \to \mathbb{R}$ and $h : [0, 1] \to \mathbb{R}$ are given functions.

Our main result is of a Bernstein–Doetsch type. Namely, we prove that if f is locally bounded from above at a point of D and rationally (h, d)-convex then it is continuous and (h, d)-convex.

Judita Dascăl: *On the equality of conjugate means of n variables* (Joint work with Zoltán Daróczy)

Let $I \subset \mathbb{R}$ be a nonvoid open interval and let $n \ge 3$ be a fixed natural number. The question is which conjugate means of n variables generated by the arithmetic mean $M\colon I^n \to I$ are weighted quasi-arithmetic means of n variables at the same time? This question is a functional equation problem:

Characterize the real valued continuous and strictly monotone functions φ, ψ defined on I and the parameters $p_1, \dots, p_n, q_1, \dots, q_n$ for which the equation

$$\varphi^{-1}\Big(\sum_{i=1}^{n} p_i\varphi(x_i) + \big(1 - \sum_{i=1}^{n} p_i\big)\varphi\big(M(x_1, \dots, x_n)\big)\Big) = \psi^{-1}\Big(\sum_{i=1}^{n} q_i\psi(x_i)\Big)$$

holds for all $x_1, ..., x_n \in I$, where

$$q_i \ge 0, \quad i = 1, \dots, n, \quad \sum_{i=1}^{n} q_i = 1,$$
$$p_j \ge 0 \text{ and } \sum_{i=1}^{n} p_i - p_j \le 1, \quad j = 1, \dots, n.$$

Włodzimierz Fechner: *On some composite functional inequalities*

We will deal with the following composite functional inequalities

$$f(f(x) - f(y)) \le f(x + y) + f(f(x - y)) - f(x) - f(y),$$
$$f(f(x) - f(y)) \le f(f(x + y)) + f(x - y) - f(x) - f(y),$$
$$f(f(x) - f(y)) \le f(f(x + y)) + f(f(x - y)) - f(f(x)) - f(y),$$

in the class of real-to-real mappings which enjoy certain regularity properties. These inequalities are motivated by some recent studies of Tomasz Kochanek and of the author concerning the following composite functional equation:

$$f(f(x)-f(y)) = f(x+y) + f(x-y) - f(x) - f(y).$$

Roman Ger: *Abstract Pythagorean Law and the corresponding functional equations*

We simplify and generalize the results of Lucio R. Berrone (cf. [1]) connected with the addition formula of the form

$$f(x+y) = f(x) + f(y) + 2f(\varphi(x,y))\,.$$

Reference

[1] Berrone L.R. *The associativity of the Pythagorean Law*, Amer. Math. Monthly **116** (2009), no. 10, 936–939.

Eszter Gselmann: *A characterization of the relative entropies* (Joint work with Gyula Maksa)

For a fixed $\alpha \in \mathbb{R}$, the sequence of functions $D_n^{\alpha}(\cdot|\cdot)\colon \Gamma_n^{\circ} \times \Gamma_n^{\circ} \to \mathbb{R}$ defined by

$$D_n^{\alpha}(p_1,\dots,p_n|q_1,\dots,q_n) = -\sum_{i=1}^{n} p_i \ln_{\alpha}\left(\frac{p_i}{q_i}\right),$$

where

$$\ln_{\alpha}(x) = \begin{cases} \frac{x^{1-\alpha}-1}{1-\alpha}, & \text{if} \quad \alpha \neq 1, \\ \ln(x), & \text{if} \quad \alpha = 1. \end{cases}$$

is called the *Shannon relative entropy* (or Kullback–Leibler entropy or Kullback's directed divergence) if $\alpha = 1$, and the *Tsallis relative entropy* if $\alpha \neq 1$, respectively.

Following the method of the basic references Aczél–Daróczy [1] and Ebanks–Sahoo–Sander [2] of investigating characterization problems of information measures, we prove a characterization theorem similar to that of [3] and [4], and we point out that the regularity conditions (say, continuity) can be avoided if $\alpha \notin \{0,1\}$, and can essentially be weakened if $\alpha \in \{0,1\}$.

References

[1] Aczél J., Daróczy Z., *On measures of information and their characterizations*, Mathematics in Science and Engineering, Vol. 115, Academic Press, New York–London, 1975.
[2] Ebanks B., Sahoo P., Sander W., *Characterizations of information measures*, World Scientific Publishing Co., Inc., River Edge, NJ, 1998.
[3] Furuichi S., *On uniqueness theorems for Tsallis entropy and Tsallis relative entropy*, IEEE Trans. Inform. Theory **51** (2005), no. 10, 3638–3645.
[4] Hobson A., *A new theorem of information theory*, J. Stat. Phys. **1** (1969), 383–391.

Michał Lewicki: *A remark on non-symmetric t-affine functions*

Let $t \in (0,1)$ be fixed. We give a solution of the functional equation

$$f(tx+(1-t)y) = tf(x)+(1-t)f(y), \tag{1}$$

where $x, y \in \mathbb{R}$ are such that $x \leqslant y$.

Furthermore, we discuss an application of the proven result in the inequality corresponding to equation (1).

Judit Makó: *Approximate convexity of Takagi type functions* (Joint work with Zsolt Páles)

Given a bounded function $\Phi\colon \mathbb{R} \to \mathbb{R}$, we define the Takagi type function $T_\Phi\colon \mathbb{R} \to \mathbb{R}$ by

$$T_\Phi(x) := \sum_{n=0}^{\infty} \frac{\Phi(2^n x)}{2^n}.$$

In this talk we give a sufficient conditions on Φ such that T_Φ is approximately Jensen-convex in the following sense

$$T_\Phi\Big(\frac{x+y}{2}\Big) \leq \frac{T_\Phi(x)+T_\Phi(y)}{2} + \Phi\Big(\frac{x-y}{2}\Big) - \Phi(0), \quad x, y \in \mathbb{R}.$$

Applications to the theory of approximately convex functions are also given.

Gyula Maksa: *Remarks on associativity*

If I is a set and $F\colon I \times I \to I$ is an associative binary operation then

$$\begin{aligned} F(F(x,y),F(u,v)) &= F(x,F(y,F(u,v))) = F(x,F(F(y,u),v)) \\ &= F(F(x,F(y,u)),v) = F(F(F(x,y),u),v) \end{aligned}$$

for all $x, y, u, v \in I$. In this talk we present the following result.

Theorem. *Let $I \subset \mathbb{R}$ be an interval and $F : I \times I \to I$ be a continuous function which is strictly monotonic in both variables. Then any of the above ten equations, with the exception of $F(x, F(F(y,u),v)) = F(F(x, F(y,u)), v)$, implies that F is associative.*

In the talk, some remarks concerning vector associativity are also formulated.

Janusz Matkowski: *A mean-value theorem*

For a function f defined in an interval I and satisfying the conditions ensuring the existence and uniqueness of the Lagrange mean $L^{[f]}$, we prove that there exists a unique two variable mean $M^{[f]}$ such that

$$\frac{f(x)-f(y)}{x-y} = M^{[f]}(f'(x), f'(y))$$

for all $x, y \in I$, $x \neq y$. The mean $M^{[f]}$ is closely related $L^{[f]}$. Necessary and sufficient condition for the equality $M^{[f]} = M^{[g]}$ is given. A family of means $\left\{\mathcal{M}^{[t]} : t \in \mathbb{R}\right\}$ relevant to the logarithmic means is introduced. The invariance of geometric mean with respect to mean-type mappings of this type is considered. A result on convergence of the sequences of iterates of some mean-type mappings and its application in solving some functional equations is given.

A counterpart of the Cauchy mean value theorem is presented. Some relations between Stolarsky means and $\mathcal{M}^{[t]}$ means are discussed.

Fruzsina Mészáros: *A generalized Steinhaus-type theorem in solving a functional equation* (Joint work with Károly Lajkó)

We give the general solution (without any regularity assumption) of equation

$$h_1\left(\frac{x}{\lambda_1(\alpha+y)}\right)\frac{1}{\lambda_1(\alpha+y)} f_Y(y) = h_2\left(\frac{y}{\lambda_2(\beta+x)}\right)\frac{1}{\lambda_2(\beta+x)} f_X(x)$$

for all $(x,y) \in \mathbb{R}_+^2$ with nonnegative functions h_1, h_2, f_X, $f_Y : \mathbb{R}_+ \to \mathbb{R}$, such that there exist sets A_1, A_2, A_3, $A_4 \subset \mathbb{R}_+$ with positive Lebesgue measure, on which these functions are positive, respectively.

We use the following generalization of Steinhaus' theorem: *Let U be an open subset of $\mathbb{R}^2$ and $F\colon U \to \mathbb{R}$ be a continuously differentiable function with nonvanishing partial derivatives, moreover let A, $B \subset \mathbb{R}$ $(A \times B \subset U)$ be measurable sets with positive Lebesgue measure, then the set $F(A,B)$ has an interior point, i.e. $F(A,B)$ contains a nonvoid open interval.*

Lajos Molnár: *Isometries and affine automorphisms of spaces of probability distribution functions*

In the paper [1] we have determined the so-called Kolmogorov–Smirnov isometries of the space $D(\mathbb{R})$ of all probability distribution functions on the real line. In this talk we present new results on the isometries of certain important subspaces of $D(\mathbb{R})$ such as the spaces of all absolute continuous, singular or discrete distribution functions. A few results on the structure of affine automorphisms of $D(\mathbb{R})$ will also be given.

Reference

[1] Dolinar G., Molnár L., *Isometries of the space of distribution functions with respect to the Kolmogorov–Smirnov metric*, J. Math. Anal. Appl. **348** (2008), 494–498.

Janusz Morawiec: *Integrable solutions of a matrix refinement equation* (Joint work with Rafał Kapica)

Assume that $(\Omega, \mathcal{A}, P)$ is a complete probability space, $L\colon \Omega \to \mathbb{R}^n$ is a random vector, $K\colon \Omega \to \mathbb{R}^{n\times n}$ and $M\colon \Omega \to \mathbb{R}^{m\times m}$ are random matrices. We investigate the existence of non-trivial integrable solutions $\varphi\colon \mathbb{R}^n \to \mathbb{R}^m$ of the following matrix refinement equation

$$\varphi(x) = \int_\Omega |\det K(\omega)| M(\omega)\varphi(K(\omega)x - L(\omega))P(d\omega).$$

Andrzej Olbryś: *On a separation for Wright convexity*

Let X will be a real linear topological space and let $D \subset X$ be an open and convex set. A function $f\colon D \to \mathbb{R}$ is said to be a Wright-convex, if

$$\bigwedge_{x,y\in D}\ \bigwedge_{\lambda\in[0,1]} f(\lambda x + (1-\lambda)y) + f((1-\lambda)x + \lambda y) \le f(x) + f(y).$$

In our talk we give sufficient and necessary conditions for separating functions g and h by a Wright convex function.

Zsolt Páles: *Regularity, monotonicity, and convexity consequences of linear functional inequalities*

Let $I \subseteq \mathbb{R}$ be a nonempty interval and denote

$$\mathcal{F}_k(I) := \{f : I^k \to \mathbb{R}\}, \quad k \in \mathbb{N}.$$

Assume that we are given a certain class of functions $\mathcal{E} \subseteq \mathcal{F}_1(I)$, a mapping $\Phi\colon \mathcal{E} \to \mathcal{F}_k$, and a subset $D \subseteq I^k$. Then the inequality

$$\Phi(f)(x_1, \dots, x_k) \geq 0, \quad (x_1, \dots, x_k) \in D, \tag{FI}$$

is called a k-variable functional inequality.

The general problem is to *describe all* $f \in \mathcal{E}$ that satisfies (FI). Another (less general) problem is to *study the common properties*, for instance, *the regularity, monotonicity and convexity properties* of the solutions of (FI).

Barbara Przebieracz: *The stability of some functional equation*

I will prove a theorem concerning the stability of the equation

$$\min\{f(x+y), f(x-y)\} = |f(x) - f(y)|$$

in the class of continuous functions $f\colon \mathbb{R} \to \mathbb{R}$.

Maciej Sablik: *On functional equations connected with MVTs*

We consider different aspects of characterizations, with use of functional equations, of some classes of mappings. Usually functional equations are stemming from Mean Value Theorems, or at least are connected to them.

Justyna Sikorska: *Stability of the Drygas functional equation*

We study the stability of the Drygas functional equation

$$f(x+y) + f(x-y) = 2f(x) + f(y) + f(-y),$$

where f maps a group into a Banach space. In our considerations we use both the direct method and the method of the invariant means.

László Székelyhidi: *Spectral analysis on commutative hypergroups* (Joint work with László Vajday)

The problem of spectral analysis is formulated on commutative hypergroups and is solved for finite dimensional varieties.

Tomasz Szostok: *Regularity of functions satisfying equations connected to quadrature rules*

Motivated by quadrature rules used in numerical analysis we consider equations of the type

$$F(y) - F(x) = (y-x)\sum_{k=1}^{n} f_k(\alpha_k x + \beta_k y). \tag{1}$$

However a quick look at the sets of the solutions of (1) in some special cases convinces us that it is hopeless to expect a full description of the solutions in the general case. The situtation changes rapidly if we know that functions f_k are continuous. In such case they are just polynomials of some degree. Therefore it is important to ask for cases where the continuity is implied by the equation itself.

Answering a problem posed by M. Sablik we prove that if equation (1) has discontinuous solutions then also equation

$$\sum_{k=1}^{n} f_k(\alpha_k x + \beta_k y) = 0$$

has discontinuous solutions. Using this result it is easy to show that in many cases equation (1) has only continuous solutions.

Wirginia Wyrobek-Kochanek: *Almost orthogonally additive functions* (Joint work with Tomasz Kochanek)

If a function f, acting on a Euclidean space $\mathbb{R}^n$, is “almost” orthogonally additive in the sense that $f(x+y) = f(x) + f(y)$ for all $(x, y) \in \perp \setminus Z$, where Z is a “negligible” subset of the $(2n-1)$-dimensional manifold $\perp \subset \mathbb{R}^{2n}$, then f coincides almost everywhere with some orthogonally additive mapping.

2. Problems and Remarks

1. Remark *to Roman Ger’s talk.* For the functional equation

$$f(x+y) = f(x) + f(y) + 2f(\varphi(x, y)) \tag{1}$$

we can obtain the following regularity result:

Theorem. *Let $\varphi\colon \mathbb{R}_+^2 \to \mathbb{R}_+$ be monotone increasing in each of its variables and $f\colon \mathbb{R}_+ \to \mathbb{R}$ be a monotone increasing function. Then f is convex (and hence it is continuous everywhere and also differentiable everywhere but at countably many points).*

Proof. Let $y \in \mathbb{R}_+$ be fixed. Then, by (1),

$$f(x+y) - f(x) = f(y) + 2f(\varphi(x, y)), \quad x \in \mathbb{R}_+.$$

The right hand side is a monotone increasing function of x, hence, for all fixed $y \in \mathbb{R}_+$, the function

$$x \mapsto f(x+y) - f(x)$$

is also increasing on $\mathbb{R}_+$. This immediately implies that f is Jensen convex. This, with the monotonicity of f, yields that f is convex on $\mathbb{R}_+$. □

Zsolt Páles

2. Remark *on the subadditivity of the Takagi function.* Motivated by the discussion of Makó's talk, we may establish the following

Theorem. *The Takagi function*

$$(*) \qquad T(x) = \sum_{n=0}^{\infty} \frac{d(2^n x, \mathbb{Z})}{2^n}, \quad x \in \mathbb{R},$$

is subadditive.

Proof. It is quite easy to verify that the mapping $x \mapsto d(x, \mathbb{Z})$, which expresses the distance of x from the closest integer, is subadditive on $\mathbb{R}$. It is also clear that the mapping $\psi(x) = a\varphi(bx)$ $(x \in \mathbb{R})$ is subadditive if $\varphi : \mathbb{R} \to \mathbb{R}$ is subadditive and a, b are fixed positive real numbers. Hence, each summand on the right hand side of $(*)$ is subadditive. Moreover, it is also clear that the finite sum of subadditive functions is subadditive as well. Finally, since the pointwise limit of a sequence of subadditive functions is subadditive, we conclude that the Takagi function, defined by the expression $(*)$, has to be subadditive. □

Roman Ger

3. Problems *concerning the generalized Takagi functions.*

Problem 1. Describe all functions $\varphi : \mathbb{R} \to \mathbb{R}$ such that the mapping

$$x \mapsto \sum_{n=0}^{\infty} \frac{\varphi(d(2^n x, \mathbb{Z}))}{2^n}$$

is smooth (i.e., continuously differentiable). We know that $\varphi(x) = ax^2 + b$ has this property for any real numbers a and b.

Problem 2. Find all real sequences (c_n) such that the mapping

$$x \mapsto \sum_{n=0}^{\infty} c_n d(2^n x, \mathbb{Z})$$

is smooth. We mention $c_n = \dfrac{1}{2^{2n+1}}$ as an example.

Zsolt Páles

4. Remark *on the first ten Debrecen–Katowice Winter Seminars on Functional Equations and Inequalities.* The 10^{th} Winter Seminar seems an appropriate occasion to list a short survey of these events:

1^{st}: Cieszyn, Poland, February 7–10, 2001
2^{nd}: Hajdúszoboszló, Hungary, January 30 – February 2, 2002
3^{rd}: Będlewo, Poland, January 29 – February 1, 2003
4^{th}: Mátraháza, Hungary, February 4-7, 2004
5^{th}: Będlewo, Poland, February 2–5, 2005
6^{th}: Berekfürdő, Hungary, February 1–4, 2006
7^{th}: Będlewo, Poland, January 31 – February 3, 2007
8^{th}: Poroszló, Hungary, January 30 – February 2, 2008
9^{th}: Będlewo, Poland, February 4–7, 2009
10^{th}: Zamárdi, Hungary, February 3–6, 2010

Participation statistics is presented in the table:

Name	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Roman Badora	×	×	×	×	×	×	×	×	×	×	10
Zoltán Boros	×	×	×	×	×	×	×	×	×	×	10
Roman Ger	×	×	×	×	×	×	×	×	×	×	10
Attila Gilányi	×	×	×	×	×	×	×	×	×	×	10
Gyula Maksa	×	×	×	×	×	×	×	×	×	×	10
Janusz Matkowski	×	×	×	×	×	×	×	×	×	×	10
Tomasz Szostok	×	×	×	×	×	×	×	×	×	×	10
Mihály Bessenyei	×	×	×	×	×	×	×	×	×		9
Zsolt Páles	×	×	×	×	×		×	×	×	×	9
Maciej Sablik	×	×	×	×	×	×	×	×		×	9
Justyna Sikorska	×		×	×	×			×	×	×	7
Zoltán Daróczy	×	×		×	×	×		×			6
Attila Házy	×	×	×	×			×		×		6
László Székelyhidi	×			×	×	×		×		×	6

Name	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Pál Burai					×	×	×	×	×	×	6
Włodzimierz Fechner					×	×	×	×	×	×	6
Janusz Morawiec					×	×	×	×	×	×	6
Karol Baron	×		×		×		×		×		5
Zoltán Kaiser	×	×	×	×	×						5
Barbara Koclęga-Kulpa	×			×		×	×		×		5
Károly Lajkóó	×	×		×	×	×					5
Zygfryd Kominek		×	×	×	×	×					5
Fruzsina Mészáros						×	×	×	×	×	5
Lech Bartłomiejczyk		×	×	×		×					4
Andrzej Olbryś			×				×	×		×	4
Barbara Przebieracz					×	×		×		×	4
Szabolcs Baják							×	×	×	×	4
Eszter Gselmann							×	×	×	×	4
László Losonczi		×	×			×					3
Grażyna Łydzińska				×		×	×				3
Wirginia Wyrobek-Kochanek								×	×	×	3
Witold Jarczyk	×		×								2
Rafał Kapica	×	×									2
Iwona Pawlikowska		×		×							2
Péter Czinder			×	×							2
Antal Járai			×			×					2
Ágota Orosz			×		×						2
Janusz Walorski			×	×							2
Gabriella Hajdu				×		×					2
Dariusz Sokołowski					×		×				2
Adrienn Varga							×		×		2
Lajos Molnár								×		×	2
Michał Lewicki									×	×	2
Tomasz Powierza		×									1
Borbála Fazekas			×								1
Zita Makó					×						1
Rezső Lovas							×				1
Agata Nowak								×			1
Tomasz Kochanek									×		1
Judit Makó										×	1
Judita Dascăl										×	1
Gergő Nagy									×		1
Peter Volkmann									×		1

Szabolcs Baják and Attila Gilányi

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