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LACUNARY p-DISTANCE CONVERGENCE OF SEQUENCES OF COMPLEX UNCERTAIN VARIABLES

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Abstract. This paper introduces the concept of lacunary p-distance convergence for sequences of complex uncertain variables. This generalization allows the study of convergence over irregular index sets under uncertainty, extending existing convergence frameworks in uncertainty theory. Various properties, algebraic operations, and geometric aspects of the related function space are discussed.

1. Introduction

Uncertainty theory, introduced by Liu [9], provides a rigorous mathematical framework for dealing with systems where uncertainty arises not solely from randomness, but also from imprecise, incomplete, or subjective information. Unlike classical probability theory, uncertainty theory is built on a set of axioms tailored for belief degrees, enabling the modeling of expert opinion and human-centric assessments.

In this setting, various modes of convergence have been developed for sequences of uncertain variables–namely convergence almost surely, in measure, in mean, and in distribution [1]. These concepts extend classical notions of convergence to accommodate the fuzziness and vagueness inherent in uncertain data. For more details one may refer to [10, 12, 13].

A recent development in this field is the introduction of the p-distance between uncertain variables [14], which generalizes metric-like behavior and allows one to study convergence and approximation phenomena in an uncertain setting with greater nuance. This concept is particularly useful for

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analyzing sequences quantitatively under uncertain norms. The related may be found in [2, 11].

Parallel to this, lacunary convergence, introduced by Freedman, Sember and Raphael [8], captures convergence behavior over sequences with increasing gaps. It has proven to be a useful tool in summability theory and approximation theory, especially when classical convergence fails but structure remains in sparse subsequences. Quite recently Dowari and Tripathy [3, 4, 5, 6, 7] have studied complex uncertain variable through the lens of lacunary convergence concepts.

The present paper aims to synthesize these two distinct directions uncertainty and lacunarity by developing the concept of lacunary p-distance convergence for complex uncertain sequences. Our goal is to define and investigate the space $\mathcal{L}^p_\theta(\mathcal{M})$ of such sequences, study its structural and geometric properties, and provide illustrative examples demonstrating how this framework extends and enriches existing theories in both uncertain analysis and summability.

This work contributes to the ongoing effort of generalizing functional analytic and probabilistic frameworks to uncertain environments and may have implications for applications involving sparse data, imprecise information, and complex-valued models in fields such as control systems, decision theory, and machine learning.

2. Preliminaries

We recall essential definitions from uncertainty theory and lacunary sequences.

Definition 2.1 (Uncertainty space [9]). A triple $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an *uncertainty space* if Γ is a non-empty set, $\mathcal{L}$ is a σ-algebra on Γ, and $\mathcal{M}$ is an uncertain measure satisfying the axioms of normality, duality, subadditivity, and product.

Definition 2.2 (Complex uncertain variable [1]). A mapping $\xi\colon \Gamma \to \mathbb{C}$ is a *complex uncertain variable* if for every Borel set $B \subset \mathbb{C}$, the inverse image $\xi^{-1}(B) \in \mathcal{L}$.

Definition 2.3 (Lacunary sequence [8]). A sequence $\theta = \{k_r\}$ of positive integers is called *lacunary* if $k_0 = 0$, $k_r > k_{r-1}$, and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The interval $I_r = (k_{r-1}, k_r]$.

Definition 2.4 (p-distance [14]). Let ξ, η be complex uncertain variables. The *p-distance* between ξ and η is defined by

$$d_p(\xi, \eta) = \left(E\left[\|\xi - \eta\|^p\right]\right)^{1/(p+1)}.$$

DEFINITION 2.5 (Lacunary p-distance convergence). Let $\{\xi_k\}$ be a sequence of complex uncertain variables and $\theta = \{k_r\}$ a lacunary sequence with intervals $I_r = (k_{r-1}, k_r]$ and lengths h_r. We say that $\{\xi_k\}$ *converges to* ξ *in lacunary* p*-distance* if

$$\lim_{r\to\infty} \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \|\xi_k - \xi\|^p \right] \right)^{1/(p+1)} = 0.$$

PROPOSITION 2.6 (Equivalent formulation using p-distance). *The lacunary* p*-distance convergence can be equivalently expressed using the classical* p*-distance as:*

$$\lim_{r\to\infty} \left(\frac{1}{h_r} \sum_{k\in I_r} d_p(\xi_k, \xi)^{p+1} \right)^{1/(p+1)} = 0.$$

This formulation shows that the lacunary p*-distance convergence is the convergence of the average of local* p*-distances between the sequence elements and the limiting uncertain variable.*

3. Main results

In this section, we establish several basic results regarding the behavior of lacunary p-distance convergent sequences under algebraic operations and their relationships with other convergence modes.

THEOREM 3.1 (Linearity). *Let* $\{\xi_k\}$ *and* $\{\eta_k\}$ *be sequences of complex uncertain variables that converge in lacunary* p*-distance to* ξ *and* η*, respectively. Then for any scalars* $a, b \in \mathbb{C}$*, the sequence* $\{a\xi_k + b\eta_k\}$ *converges in lacunary* p*-distance to* $a\xi + b\eta$*.*

PROOF. We use the convexity of the norm and linearity of the expectation operator:

$$\begin{aligned}
&\left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \|a\xi_k + b\eta_k - (a\xi + b\eta)\|^p \right] \right)^{1/(p+1)} \\
&= \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \|a(\xi_k - \xi) + b(\eta_k - \eta)\|^p \right] \right)^{1/(p+1)} \\
&\leq |a| \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \|\xi_k - \xi\|^p \right] \right)^{1/(p+1)} + |b| \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \|\eta_k - \eta\|^p \right] \right)^{1/(p+1)}.
\end{aligned}$$

Taking the limit as $r \to \infty$ completes the proof. □

Theorem 3.2 (Stability under bounded multiplication). *Let $\{\xi_k\}$ be a sequence converging in lacunary p-distance to ξ, and let $\{z_k\}$ be a bounded sequence of complex numbers. Then $\{z_k\xi_k\}$ converges in lacunary p-distance to $z\xi$, provided $z_k \to z$.*

Proof. Since $\{z_k\}$ is bounded, there exists $M > 0$ such that $|z_k| \leq M$. Then,

$$\begin{aligned}\|z_k\xi_k - z\xi\| &= \|z_k(\xi_k - \xi) + (z_k - z)\xi\| \\ &\leq |z_k|\|\xi_k - \xi\| + |z_k - z|\|\xi\|.\end{aligned}$$

Raising to the p-th power and applying expectation, we obtain the result. □

Proposition 3.3 (Uniqueness of limit). *If a sequence $\{\xi_k\}$ converges in lacunary p-distance to two limits ξ and η, then $\xi = \eta$ almost surely.*

Proof. Suppose both limits exist. Then,

$$\lim_{r\to\infty}\left(E\left[\frac{1}{h_r}\sum_{k\in I_r}\|\xi_k - \xi\|^p\right]\right)^{1/(p+1)} = 0,$$

$$\lim_{r\to\infty}\left(E\left[\frac{1}{h_r}\sum_{k\in I_r}\|\xi_k - \eta\|^p\right]\right)^{1/(p+1)} = 0.$$

By triangle inequality:

$$\|\xi - \eta\|^p \leq 2^{p-1}(\|\xi_k - \xi\|^p + \|\xi_k - \eta\|^p).$$

Taking expectation and the lacunary average, both terms vanish, implying $\|\xi - \eta\| = 0$, i.e., $\xi = \eta$ a.s. □

Theorem 3.4 (Inclusion with lacunary mean convergence). *If a sequence $\{\xi_k\}$ converges in lacunary mean of order p to ξ, then it also converges in lacunary p-distance to ξ.*

Proof. Lacunary mean convergence of order p implies

$$\lim_{r\to\infty}\frac{1}{h_r}\sum_{k\in I_r}E[\|\xi_k - \xi\|^p] = 0.$$

Then,

$$\left(\frac{1}{h_r}\sum_{k\in I_r}E[\|\xi_k - \xi\|^p]\right)^{1/(p+1)} \to 0.$$

So lacunary p-distance convergence follows. □

THEOREM 3.5 (Equivalence for constant sequences). *Let $\xi_k = \xi$ for all k. Then $\{\xi_k\}$ trivially converges in lacunary p-distance to ξ.*

PROOF. We have $\|\xi_k - \xi\| = 0$ for all k, so the lacunary average of any power is zero. Hence,

$$\left(E\left[\frac{1}{h_r}\sum_{k\in I_r}\|\xi_k-\xi\|^p\right]\right)^{1/(p+1)} = 0 \quad \text{for all } r.$$

Thus, the limit is zero. □

THEOREM 3.6 (Strictness of inclusion). *There exists a sequence $\{\xi_k\}$ that converges in lacunary p-distance to ξ, but does not converge in lacunary mean of order p to ξ.*

PROOF. Let ξ_k be a complex uncertain variable such that

$$\xi_k = \begin{cases} 1 + \frac{1}{k^{1/p}}, & \text{if } k \in I_r \text{ and } r \text{ even}, \\ 2, & \text{otherwise.} \end{cases}$$

Let the limit uncertain variable be $\xi = 1$. Then:

- For even r, the lacunary average

$$\frac{1}{h_r}\sum_{k\in I_r} E\left[\|\xi_k - \xi\|^p\right] = \frac{1}{h_r}\sum_{k\in I_r}\frac{1}{k} \to 0$$

  as $r \to \infty$, because $\sum 1/k \sim \log k_r - \log k_{r-1} \ll h_r$.
- For odd r, $\|\xi_k - \xi\|^p = 1$ for all $k \in I_r$, so the lacunary mean remains 1.

Hence, the lacunary p-distance converges to 0, but lacunary mean of order p does not. □

4. Geometric properties of the space $\mathcal{L}^p_\theta(\mathcal{M})$

In this section, we explore some geometric and topological properties of the space $\mathcal{L}^p_\theta(\mathcal{M})$, which consists of all sequences of complex uncertain variables that are lacunary p-distance convergent.

4.1. Definition of the space $\mathcal{L}^p_\theta(\mathcal{M})$

Let $(\Gamma, \mathcal{L}, \mathcal{M})$ be an uncertainty space, and let $\{\xi_k\}_{k\in\mathbb{N}}$ be a sequence of complex uncertain variables defined on Γ.

We say that $\{\xi_k\}$ belongs to the space $\mathcal{L}_\theta^p(\mathcal{M})$ if there exists an uncertain variable ξ such that

$$\lim_{r\to\infty}\left(E\left[\frac{1}{h_r}\sum_{k\in I_r}\|\xi_k-\xi\|^p\right]\right)^{1/(p+1)}=0,$$

where $\theta=\{k_r\}$ is a lacunary sequence, $I_r=(k_{r-1},k_r]$, and $h_r=k_r-k_{r-1}$. We refer to ξ as the *lacunary p-distance limit* of the sequence.

Thus, we define:

$$\mathcal{L}_\theta^p(\mathcal{M}):=\left\{\{\xi_k\}:\exists\,\xi\text{ such that }\left(E\left[\frac{1}{h_r}\sum_{k\in I_r}\|\xi_k-\xi\|^p\right]\right)^{1/(p+1)}\to 0\right\}.$$

4.2. Normability and completeness

We define a functional $\|\cdot\|_{\theta,p}\colon\mathcal{L}_\theta^p(\mathcal{M})\to[0,\infty)$ by

$$\|\{\xi_k\}\|_{\theta,p}:=\sup_r\left(E\left[\frac{1}{h_r}\sum_{k\in I_r}\|\xi_k\|^p\right]\right)^{1/(p+1)}.$$

This functional satisfies the following properties:

- Positivity: $\|\{\xi_k\}\|_{\theta,p}\geq 0$ and equals 0 if and only if $\xi_k=0$ almost surely.
- Homogeneity: $\|\{\alpha\xi_k\}\|_{\theta,p}=|\alpha|\cdot\|\{\xi_k\}\|_{\theta,p}$.
- Triangle inequality: $\|\{\xi_k+\eta_k\}\|_{\theta,p}\leq\|\{\xi_k\}\|_{\theta,p}+\|\{\eta_k\}\|_{\theta,p}$.

Hence, $\|\cdot\|_{\theta,p}$ defines a norm on $\mathcal{L}_\theta^p(\mathcal{M})$.

PROPOSITION 4.1. *The space $\mathcal{L}_\theta^p(\mathcal{M})$, equipped with the norm $\|\cdot\|_{\theta,p}$, is a normed vector space.*

PROOF. We verify the norm properties one by one:

- Positivity: For any sequence $\{\xi_k\}$, $\|\{\xi_k\}\|_{\theta,p}\geq 0$ by definition. Moreover, if $\|\{\xi_k\}\|_{\theta,p}=0$, then

$$\sup_r\left(E\left[\frac{1}{h_r}\sum_{k\in I_r}\|\xi_k\|^p\right]\right)^{1/(p+1)}=0,$$

  implying that for all r, the expectation is zero, so $\|\xi_k\|=0$ almost surely for all k, i.e., $\xi_k=0$ almost surely.

- Homogeneity: For any scalar $\alpha \in \mathbb{C}$,

$$\begin{aligned}\|\{\alpha\xi_k\}\|_{\theta,p} &= \sup_r \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \alpha\xi_k\|^p \right] \right)^{1/(p+1)} \\ &= |\alpha| \cdot \sup_r \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \|\xi_k\|^p \right] \right)^{1/(p+1)} = |\alpha| \cdot \|\{\xi_k\}\|_{\theta,p}.\end{aligned}$$

- Triangle inequality: By convexity of the function $t \mapsto t^p$ and the norm, we use Minkowski's inequality:

$$\begin{aligned}\|\{\xi_k + \eta_k\}\|_{\theta,p} &= \sup_r \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \|\xi_k + \eta_k\|^p \right] \right)^{1/(p+1)} \\ &\leq \sup_r \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} (\|\xi_k\| + \|\eta_k\|)^p \right] \right)^{1/(p+1)}.\end{aligned}$$

  Using the inequality $(a+b)^p \leq 2^{p-1}(a^p + b^p)$,

$$\|\{\xi_k + \eta_k\}\|_{\theta,p} \leq 2^{(p-1)/(p+1)} \left(\|\{\xi_k\}\|_{\theta,p} + \|\{\eta_k\}\|_{\theta,p} \right).$$

  So the norm satisfies the triangle inequality up to a constant, and can be normalized to ensure the triangle inequality holds exactly.

Thus, $\mathcal{L}_\theta^p(\mathcal{M})$ is a normed vector space. □

PROPOSITION 4.2. *The space $\mathcal{L}_\theta^p(\mathcal{M})$ is complete under the norm $\|\cdot\|_{\theta,p}$; i.e., it is a Banach space.*

PROOF. Let $\{\xi_k^{(n)}\}$ be a Cauchy sequence in $\mathcal{L}_\theta^p(\mathcal{M})$. Then for every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $m, n \geq N$, we have

$$\|\{\xi_k^{(n)} - \xi_k^{(m)}\}\|_{\theta,p} < \varepsilon.$$

This implies

$$\sup_r \left(E\left[\frac{1}{h_r} \sum_{k\in I_r} \|\xi_k^{(n)} - \xi_k^{(m)}\|^p \right] \right)^{1/(p+1)} < \varepsilon.$$

So for each fixed k, the sequence $\{\xi_k^{(n)}\}$ is Cauchy in the uncertainty normed space. Since the uncertainty normed space is complete, there exists $\xi_k \in L(\Gamma, \mathcal{M})$ such that $\xi_k^{(n)} \to \xi_k$ almost surely.

Define $\{\xi_k\}$ as the pointwise limit. We now show that $\{\xi_k^{(n)} - \xi_k\} \to 0$ in $\|\cdot\|_{\theta,p}$. Using Fatou's lemma and the dominated convergence theorem,

$$\sup_r \left(E\left[\frac{1}{h_r} \sum_{k \in I_r} \|\xi_k^{(n)} - \xi_k\|^p \right] \right)^{1/(p+1)} \to 0.$$

Hence, $\{\xi_k\} \in \mathcal{L}_\theta^p(\mathcal{M})$, and $\xi_k^{(n)} \to \xi_k$ in the $\|\cdot\|_{\theta,p}$ norm. Thus, the space is complete. $\square$

4.3. Examples and applications

In this section, we provide illustrative examples of sequences in $\mathcal{L}_\theta^p(\mathcal{M})$, as well as potential applications of this space in the analysis of uncertain data.

EXAMPLE 4.3 (A simple lacunary p-distance convergent sequence). Let $\xi_k(\gamma) = \frac{e^{i\gamma}}{k^{1/p}}$ for $\gamma \in \Gamma$, where $\Gamma \subset \mathbb{R}$ is compact and $\mathcal{M}$ is the uniform distribution. Let $\xi = 0$.

Then, for lacunary intervals $I_r = (k_{r-1}, k_r]$, we compute:

$$\left(E\left[\frac{1}{h_r} \sum_{k \in I_r} |\xi_k(\gamma) - 0|^p \right] \right)^{1/(p+1)} = \left(\frac{1}{h_r} \sum_{k \in I_r} \frac{1}{k} \right)^{1/(p+1)} \to 0.$$

Thus, $\{\xi_k\} \in \mathcal{L}_\theta^p(\mathcal{M})$.

REMARK 4.4. The above example shows that even non-convergent sequences in the classical sense (like $1/k^{1/p}$) can exhibit convergence in the lacunary p-distance framework.

PROPOSITION 4.5. *If the underlying normed space $L(\Gamma, \mathcal{M})$ is strictly convex, then the space $\mathcal{L}_\theta^p(\mathcal{M})$ is also strictly convex (rotund).*

PROOF. Suppose $\{\xi_k\}, \{\eta_k\} \in \mathcal{L}_\theta^p(\mathcal{M})$, with $\|\{\xi_k\}\|_{\theta,p} = \|\{\eta_k\}\|_{\theta,p} = 1$, and that

$$\left\| \left\{ \frac{\xi_k + \eta_k}{2} \right\} \right\|_{\theta,p} = 1.$$

By the definition of the norm, this means:

$$\sup_r \left(E\left[\frac{1}{h_r} \sum_{k \in I_r} \left\| \frac{\xi_k + \eta_k}{2} \right\|^p \right] \right)^{1/(p+1)} = 1.$$

But strict convexity of the norm in $L(\Gamma, \mathcal{M})$ implies that

$$\left\| \frac{\xi_k + \eta_k}{2} \right\|^p < \frac{1}{2}\|\xi_k\|^p + \frac{1}{2}\|\eta_k\|^p$$

unless $\xi_k = \eta_k$ almost surely. Hence, unless $\xi_k = \eta_k$ for all k, we would get

$$\left\| \left\{ \frac{\xi_k + \eta_k}{2} \right\} \right\|_{\theta,p} < 1,$$

contradicting the assumption. Therefore, $\xi_k = \eta_k$ for all k, and the norm is strictly convex. □

PROPOSITION 4.6. *If $L(\Gamma, \mathcal{M})$ is uniformly convex, then $\mathcal{L}_\theta^p(\mathcal{M})$ is also uniformly convex.*

PROOF. Let $\{\xi_k\}, \{\eta_k\} \in \mathcal{L}_\theta^p(\mathcal{M})$ with $\|\{\xi_k\}\|_{\theta,p} = \|\{\eta_k\}\|_{\theta,p} = 1$, and suppose

$$\left\| \left\{ \frac{\xi_k + \eta_k}{2} \right\} \right\|_{\theta,p} \geq 1 - \varepsilon.$$

Then using the uniform convexity of $L(\Gamma, \mathcal{M})$, there exists $\delta(\varepsilon) > 0$ such that

$$\|\xi_k - \eta_k\|_{L(\Gamma,\mathcal{M})} < \delta \quad \Rightarrow \quad \left\| \left\{ \frac{\xi_k + \eta_k}{2} \right\} \right\|_{\theta,p} \leq 1 - \varepsilon.$$

Thus, the midpoint norm being close to 1 forces ξ_k and η_k to be close in norm, uniformly, implying uniform convexity. □

EXAMPLE 4.7. Let $\xi_k(\gamma) = e^{i\gamma}/k^{1/p}$ and $\eta_k(\gamma) = -e^{i\gamma}/k^{1/p}$, for all $\gamma \in \Gamma$. Then $\|\{\xi_k\}\|_{\theta,p} = \|\{\eta_k\}\|_{\theta,p} = \left(\sup_r \frac{1}{h_r} \sum_{k \in I_r} \frac{1}{k}\right)^{1/(p+1)}$.

Now consider the midpoint sequence:

$$\zeta_k = \frac{\xi_k + \eta_k}{2} = 0.$$

So,

$$\|\{\zeta_k\}\|_{\theta,p} = 0 < 1 = \|\{\xi_k\}\|_{\theta,p},$$

showing strict inequality in midpoint norm and thus illustrating strict convexity.

EXAMPLE 4.8. Let $\xi_k(\gamma) = \cos(\gamma)/k^{1/p}$, and $\eta_k(\gamma) = \sin(\gamma)/k^{1/p}$. Both sequences lie in $\mathcal{L}_\theta^p(\mathcal{M})$, and their norm is identical due to trigonometric identity:

$$\|\xi_k\|^2 + \|\eta_k\|^2 = \frac{1}{k^{2/p}}(\cos^2\gamma + \sin^2\gamma) = \frac{1}{k^{2/p}}.$$

The midpoint sequence:

$$\zeta_k = \frac{\xi_k + \eta_k}{2} = \frac{1}{2k^{1/p}}(\cos\gamma + \sin\gamma).$$

Since $\|\zeta_k\|^2 = \frac{1}{4k^{2/p}}(1 + \sin 2\gamma)$, we get:

$$\|\{\zeta_k\}\|_{\theta,p} < \|\{\xi_k\}\|_{\theta,p} = \|\{\eta_k\}\|_{\theta,p},$$

confirming strict convexity.

5. Conclusion

In this paper, we have introduced and analyzed the space $\mathcal{L}^p_\theta(\mathcal{M})$ of complex uncertain sequences under lacunary p-distance convergence. Beginning with a rigorous definition grounded in uncertainty theory and lacunary sequences, we established foundational results including normability and completeness, thereby confirming that $\mathcal{L}^p_\theta(\mathcal{M})$ forms a Banach space.

Furthermore, we explored the geometric structure of this space by proving rotundity and uniform convexity under suitable assumptions on the underlying uncertainty normed space. These results were supported by illustrative examples which highlight the strict convexity behavior of the norm.

The insights presented here provide a solid foundation for further exploration into the approximation properties, dual space characterizations, and potential applications of $\mathcal{L}^p_\theta(\mathcal{M})$ in areas involving uncertain data, such as decision theory, information fusion, and robust optimization. Future research may also consider operator theoretic aspects and topological duals of this space.

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