

FIXED POINT THEOREMS FOR TWO PAIRS OF MAPPINGS SATISFYING A NEW TYPE OF COMMON LIMIT RANGE PROPERTY IN G_p METRIC SPACES

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Abstract. The purpose of this paper is to prove a general fixed point theorem for mappings involving almost altering distances and satisfying a new type of common limit range property in G_p metric spaces. In the last part of the paper, some fixed point results for mappings satisfying contractive conditions of integral type and for φ -contractive mappings are obtained.

1. Introduction

Let (X, d) be a metric space and S, T be self mappings of X . In [19], Jungck defined S and T to be compatible if

$$\lim_{n \rightarrow \infty} d(STx_n, TSx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = t$$

for some $t \in X$.

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This concept has been frequently used to prove existence theorems in fixed point theory.

Let f, g be self maps of a nonempty set X . A point $x \in X$ is a coincidence point of f and g if $w = fx = gx$; w is then said to be a point of coincidence of f and g . The set of all coincidence points of f and g is denoted by $\mathcal{C}(f, g)$.

In 1994, Pant ([35]) introduced the notion of pointwise R -weakly commuting mappings. It is proved in [36] that the pointwise R -weakly commutativity is equivalent to commutativity in coincidence points.

In [20], Jungck introduced the concept of weakly compatible mappings.

DEFINITION 1.1 ([20]). Let X be a nonempty set and f, g be self mappings on X . Functions f and g are *weakly compatible* if $fgu = gfu$ for all $u \in \mathcal{C}(f, g)$.

Hence, f and g are weakly compatible if and only if f and g are pointwise R -weakly commuting.

The study of common fixed points for noncompatible mappings is also interesting, the work in this regard was initiated by Pant [37]–[39].

Aamri and El Moutawakil introduced a generalization of noncompatible mappings in [1].

DEFINITION 1.2 ([1]). Let S and T be two self mappings of a metric space (X, d) . We say that S and T satisfy *(E.A)-property* if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = t$$

for some $t \in X$.

REMARK 1.1. It is clear that two self mappings S and T of a metric space (X, d) are noncompatible if there exists $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = t$ for some $t \in X$ but $\lim_{n \rightarrow \infty} d(STx_n, TSx_n)$ is nonzero or does not exist. Therefore, two noncompatible self mappings of a metric space (X, d) satisfy *(E.A)-property*.

It is known from [41, 43] that the notions of weakly compatible mappings and mappings satisfying *(E.A)-property* are independent.

In 2005, Liu et al. ([26]) defined the notion of common *(E.A)-property*.

DEFINITION 1.3 ([26]). Two pairs (A, S) and (B, T) of self mappings on a metric space (X, d) are said to satisfy common *(E.A)-property* if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = t$$

for some $t \in X$.

There exists a vast literature concerning the study of fixed points for mappings satisfying $(E.A)$ -property.

In 2011, Sintunavarat and Kumam ([58]) introduced the notion of common limit range property.

DEFINITION 1.4 ([58]). A pair (A, S) of self mappings of a metric space (X, d) is said to satisfy the *common limit range property with respect to S* (shortly $CLR_{(S)}$ -property), if there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$ for some $t \in S(X)$.

Thus, we can infer that a pair (A, S) satisfying $(E.A)$ -property along with the closedness of the subspace $S(X)$ always has $CLR_{(S)}$ -property with respect to S .

Recently, Imdad et al. ([16]) introduced the notion of common limit range property for two pairs of self mappings.

DEFINITION 1.5 ([17]). Two pairs (A, S) and (B, T) of self mappings in a metric space (X, d) are said to satisfy *common limit range property with respect to S and T* (shortly $CLR_{(S,T)}$ -property), if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = t$ for some $t \in S(X) \cap T(X)$.

Some fixed point results for pairs of mappings with $CLR_{(S)}$ and $CLR_{(S,T)}$ -property are obtained in [15, 17, 18, 23] and in other papers.

Now we introduce a new type of common limit range property.

DEFINITION 1.6 ([45]). Let A, S and T be self mappings of a metric space (X, d) . The pair (A, S) is said to satisfy *limit range property with respect to T* (shortly $CLR_{(A,S)T}$ -property), if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$$

for some $t \in S(X) \cap T(X)$.

EXAMPLE 1.1. Let \mathbb{R}_+ be the metric space with the usual metric, $Ax = \frac{x^2+1}{2}$, $Sx = \frac{x+1}{2}$, $Tx = x + \frac{1}{4}$. Then $S(X) = [\frac{1}{2}, \infty)$, $T(X) = [\frac{1}{4}, \infty)$, $S(X) \cap T(X) = [\frac{1}{2}, \infty)$. Let $\{x_n\}$ be a sequence in X such that $\lim_{n \rightarrow \infty} x_n = 0$. Then $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \frac{1}{2} = z$ and $z \in S(X) \cap T(X)$.

REMARK 1.2. Let A, B, S and T satisfy the common limit range property with respect to S and T . Then (A, S) satisfy the common limit range property with respect to T . The converse is not true. To see this, consider the metric space and the functions A, S, T defined in Example 1.1 and put $Bx = x^2 + \frac{1}{4}$.

Suppose that (A, S) and (B, T) have $CLR_{(S,T)}$ -property. Then there exist sequences $\{x_n\}, \{y_n\}$ and $t \geq \frac{1}{2}$ such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = t$. Since $\lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = t \geq \frac{1}{2}$, then $y_n \rightarrow 1$ and thus, $t = \frac{5}{4}$. But $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n$ only if $x_n \rightarrow 0$ or $x_n \rightarrow 1$. In both cases $\lim_{n \rightarrow \infty} Ax_n \neq \frac{5}{4}$, a contradiction. Hence, the pairs (A, S) and (B, T) do not satisfy $CLR_{(S,T)}$ -property.

DEFINITION 1.7 ([24]). An *altering distance* is a mapping $\psi: [0, \infty) \rightarrow [0, \infty)$ such that

- (ψ_1) ψ is increasing and continuous,
- (ψ_2) $\psi(t) = 0$ if and only if $t = 0$.

Fixed point problems involving altering distances have been studied in [47, 55, 56] and in other papers.

DEFINITION 1.8 ([51]). A function $\psi: [0, \infty) \rightarrow [0, \infty)$ is an *almost altering distance* if

- (ψ_1) ψ is continuous,
- (ψ_2) $\psi(t) = 0$ if and only if $t = 0$.

EXAMPLE 1.2. Every altering distance is an almost altering distance, but the converse is not true as we can see in the following example of function:

$$\psi(t) = \begin{cases} t, & t \in [0, 1], \\ \frac{1}{t}, & t \in (1, \infty). \end{cases}$$

2. Preliminaries

In [11, 12], Dhage introduced a new class of generalized metric spaces, named D -metric spaces.

Mustafa and Sims ([32, 33]) proved that most of the claims concerning the fundamental topological structures on D -metric spaces are incorrect and introduced an appropriate notion of generalized metric space, named G -metric space. In fact, Mustafa, Sims and other authors studied many fixed point results for self mappings under certain conditions (see [29, 30, 31, 32, 34, 57] and other papers).

DEFINITION 2.1 ([33]). Let X be a nonempty set and let $G: X^3 \rightarrow \mathbb{R}_+$ be a function satisfying the following properties:

- (G_1) $G(x, y, z) = 0$ if $x = y = z$,
 (G_2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
 (G_3) $G(x, y, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,
 (G_4) $G(x, y, z) = G(y, z, x) = \dots$ (symmetry in all three variables),
 (G_5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (triangle inequality).

The function G is called a G -metric on X and (X, G) is called a G -metric space.

Note that if $G(x, y, z) = 0$ then $x = y = z$.

REMARK 2.1. Let (X, G) be a G -metric space. If $y = z$, then by [45, Lemma 5.1], $G(x, y, y)$, is a quasi-metric on X . Hence, (X, Q) , where $Q(x, y) = G(x, y, y)$, is a quasi-metric space and since every metric space is a particular case of a quasi-metric space, it follows that the notion of G -metric space is a generalization of metric space.

In 1994, Matthews ([28]) introduced the concept of partial metric spaces as a part of the study of denotational semantics of dataflow networks and proved the Banach contraction principle in such spaces.

Many authors studied some fixed points for mappings satisfying contractive conditions in partial metric spaces.

Quite recently, in [4, 9, 10, 21, 22], some fixed point theorems under various contractive conditions in partial metric spaces have been proved.

DEFINITION 2.2 ([28]). Let X be a nonempty set. A function $p: X \times X \rightarrow \mathbb{R}_+$ is said to be a *partial metric on X* if, for all $x, y, z \in X$:

- (P_1) $p(x, x) = p(y, y) = p(x, y)$ if and only if $x = y$,
 (P_2) $p(x, x) \leq p(x, y)$,
 (P_3) $p(x, y) = p(y, x)$,
 (P_4) $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$.

The pair (X, p) is called a *partial metric space*.

REMARK 2.2. Obviously, every metric space is a partial metric space.

Quite recently, Ahmadi Zand and Dehghan Nezhad ([3]) introduced a generalization of a G -metric space and a partial metric space, named G_p -metric space. Some results on fixed points in G_p -metric space have been obtained, e.g., in [5]–[7], [40, 52, 53].

DEFINITION 2.3 ([3, 40]). Let X be a nonempty set. A function $G_p: X^3 \rightarrow \mathbb{R}_+$ is called a G_p -metric on X if the following conditions are satisfied:

- (GP_1) $x = y = z$ if $G_p(x, y, z) = G_p(x, x, x) = G_p(y, y, y) = G_p(z, z, z)$,

(GP₂) $0 \leq G_p(x, x, x) \leq G_p(x, x, y) \leq G_p(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,

(GP₃) $G_p(x, y, z) = G_p(y, z, x) = \dots$ (symmetry in all three variables),

(GP₄) $G_p(x, y, z) \leq G_p(x, a, a) + G_p(a, y, z) - G_p(a, a, a)$ for all $x, y, z, a \in X$.

The pair (X, G_p) is called a G_p -metric space.

LEMMA 2.1 ([3]). Let G_p be a G_p -metric on a nonempty set X . Then $G_p(x, y, y) \leq 2G_p(x, x, y) - G_p(x, x, x)$.

LEMMA 2.2 ([5]). Let (X, G_p) be a G_p -metric space. Then:

- 1) if $G_p(x, y, z) = 0$ then $x = y = z$,
- 2) if $x \neq y$ then $G_p(x, y, y) > 0$.

DEFINITION 2.4 ([3]). Let (X, G_p) be a G_p -metric space and let $\{x_n\}$ be a sequence of points in X . A point $x \in X$ is said to be the *limit of the sequence* $\{x_n\}$, denoted by $x_n \rightarrow x$, if $\lim_{n, m \rightarrow \infty} G_p(x, x_n, x_m) = G_p(x, x, x)$. Then the sequence $\{x_n\}$ is called G_p -convergent to x .

LEMMA 2.3 ([3]). Let (X, G_p) be a G_p -metric space. Then, for any $\{x_n\} \subset X$ and $x \in X$, the following properties are equivalent:

- a) $\{x_n\}$ is G_p -convergent to x ,
- b) $G_p(x_n, x_n, x) \rightarrow G_p(x, x, x)$ as $n \rightarrow \infty$,
- c) $G_p(x_n, x, x) \rightarrow G_p(x, x, x)$ as $n \rightarrow \infty$.

LEMMA 2.4 ([40]). If $x_n \rightarrow x$ in a G_p -metric space (X, G_p) and $G_p(x, x, x) = 0$, then for every $y \in Y$

- 1) $\lim_{n \rightarrow \infty} G_p(x_n, y, y) = G_p(x, y, y)$,
- 2) $\lim_{n \rightarrow \infty} G_p(x_n, x_n, y) = G_p(x, x, y)$.

DEFINITION 2.5 ([45]). Let A, S and T be self mappings of a G_p -metric space (X, G_p) . Then the pair (A, S) is said to satisfy the *common limit range property with respect to T* (shortly $CLR_{(A,S)T}$ -property), if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$$

for some $z \in X$ with $G_p(z, z, z) = 0$ and $z \in S(X) \cap T(X)$.

3. Implicit relations

Several classical fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit function in [43, 44].

A new type of implicit relation has been introduced in [4].

Some fixed point theorems for mappings satisfying an implicit relation in G -metric spaces have been proved in [48]–[51].

Recently, fixed point results for mappings satisfying an implicit relation in partial metric spaces have been obtained in [13, 14, 59].

Quite recently, some fixed point results for mappings satisfying implicit relations in G_p -metric spaces have been obtained in [52, 53].

DEFINITION 3.1 ([4]). Let \mathcal{F}_{G_p} be the set of all real continuous functions $F: \mathbb{R}_+^6 \rightarrow \mathbb{R}$ satisfying the conditions:

$$(F_1) \quad F(t, 0, t, 0, 0, t) > 0, \quad \forall t > 0,$$

$$(F_2) \quad F(t, 0, 0, t, t, 0) > 0, \quad \forall t > 0,$$

$$(F_3) \quad F(t, t, 0, 0, t, t) > 0, \quad \forall t > 0.$$

EXAMPLE 3.1. $F(t_1, \dots, t_6) = t_1 - at_2 - bt_3 - ct_4 - dt_5 - et_6$, where $a, b, c, d, e \in \mathbb{R}$ and $\max\{b + e, c + d, a + d + e\} < 1$.

EXAMPLE 3.2. $F(t_1, \dots, t_6) = t_1 - k \max\{t_2, t_3, \dots, t_6\}$, where $k \in [0, 1)$.

EXAMPLE 3.3. $F(t_1, \dots, t_6) = t_1 - k \max\{t_2, t_3, t_4, \frac{t_5 + t_6}{2}\}$, where $k \in [0, 1)$.

EXAMPLE 3.4. $F(t_1, \dots, t_6) = t_1 - k \max\{t_2, \frac{t_3 + t_4}{2}, \frac{t_5 + t_6}{2}\}$, where $k \in [0, 1)$.

EXAMPLE 3.5. $F(t_1, \dots, t_6) = t_1 - at_2 - b \max\{t_3, t_4\} - c \max\{t_2, t_5, t_6\}$, where $a, b, c \in \mathbb{R}$ and $\max\{a + c, b + c\} < 1$.

EXAMPLE 3.6. $F(t_1, \dots, t_6) = t_1 - \max\{ct_2, ct_3, ct_4, at_5 + bt_6\}$, where $a, b, c \in \mathbb{R}$ and $\max\{a, b, c, a + b\} < 1$.

EXAMPLE 3.7. $F(t_1, \dots, t_6) = t_1^2 - at_2^2 - b \max\{t_3t_4, t_5t_6\}$, where $a, b \in \mathbb{R}$ and $a + b < 1$.

EXAMPLE 3.8. $F(t_1, \dots, t_6) = t_1 - at_2 - b\sqrt{t_3t_4} - c\sqrt{t_5t_6}$, where $a, b, c \in \mathbb{R}$ and $a + c < 1$.

The purpose of this paper is to prove a general fixed point theorem for mappings involving almost altering distance and a new type of common limit range property in G_p -metric spaces. In the last part of the paper, some fixed point results for mappings satisfying contractive conditions of integral type and for φ -contractive mappings are obtained.

4. Main results

LEMMA 4.1 ([2]). *Let f, g be two weakly compatible self mappings of a nonempty set X . If f and g have a unique point of coincidence $w = fx = gx$ for some $x \in X$, then w is the unique common fixed point of f and g .*

THEOREM 4.1. *Let A, B, S and T be self mappings of a G_p -metric space (X, G_p) satisfying*

$$(4.1) \quad F \left(\begin{array}{l} \psi(G_p(Ax, By, By)), \psi(G_p(Sx, Ty, Ty)), \psi(G_p(Sx, Sx, Ax)), \\ \psi(G_p(Ty, By, By)), \psi(G_p(Sx, By, By)), \psi(G_p(Ax, Ty, Ty)) \end{array} \right) \leq 0$$

for all $x, y \in X$, where $F \in \mathcal{F}_{G_p}$ and ψ is an almost altering distance.

If (A, S) and T satisfy $CLR_{(A,S)T}$ -property, then

- 1) $\mathcal{C}(A, S) \neq \emptyset$,
- 2) $\mathcal{C}(B, T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point z and $G_p(z, z, z) = 0$.

PROOF. Since (A, S) and T satisfy $CLR_{(A,S)T}$ -property, then there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ with $G_p(z, z, z) = 0$ and $z \in S(X) \cap T(X)$.

Since $z \in T(X)$, there exists $u \in X$ such that $z = Tu$. By (4.1) we obtain

$$(4.2) \quad F \left(\begin{array}{l} \psi(G_p(Ax_n, Bu, Bu)), \psi(G_p(Sx_n, Tu, Tu)), \psi(G_p(Sx_n, Sx_n, Ax_n)), \\ \psi(G_p(Tu, Bu, Bu)), \psi(G_p(Sx_n, Bu, Bu)), \psi(G_p(Ax_n, Tu, Tu)) \end{array} \right) \leq 0.$$

Since $G_p(Ax_n, Sx_n, Sx_n) \leq G_p(Ax_n, z, z) + G_p(z, Sx_n, Sx_n)$, by Lemma 2.4

$$\lim_{n \rightarrow \infty} G_p(Ax_n, Sx_n, Sx_n) \leq G_p(z, z, z) + G_p(z, z, z) = 0.$$

Letting n tend to infinity in (4.2) we obtain

$$F\left(\begin{matrix} \psi(G_p(z, Bu, Bu)), 0, 0, \\ \psi(G_p(z, Bu, Bu)), \psi(G_p(z, Bu, Bu)), 0 \end{matrix}\right) \leq 0,$$

which contradicts (F_2) , if $G_p(z, Bu, Bu) > 0$. Hence, $G_p(z, Bu, Bu) = 0$ and by Lemma 2.2 (1), $z = Bu = Tu$. Therefore, $\mathcal{C}(B, T) \neq \emptyset$ and $G_p(z, z, z) = 0$. Since $z \in S(X)$, there exists $v \in X$ such that $z = Sv$.

By (4.1) we obtain

$$F\left(\begin{matrix} \psi(G_p(Av, Bu, Bu)), \psi(G_p(Sv, Tu, Tu)), \psi(G_p(Sv, Sv, Av)), \\ \psi(G_p(Tu, Bu, Bu)), \psi(G_p(Sv, Bu, Bu)), \psi(G_p(Av, Tu, Tu)) \end{matrix}\right) \leq 0,$$

$$F\left(\begin{matrix} \psi(G_p(Av, z, z)), 0, \psi(G_p(Av, z, z)), \\ 0, 0, \psi(G_p(Av, z, z)) \end{matrix}\right) \leq 0,$$

which contradicts (F_1) , if $G_p(Av, z, z) > 0$. Hence, $G_p(Av, z, z) = 0$ and by Lemma 2.2 (1), $z = Av = Sv$. Therefore, $z = Av = Sv = Tu = Bu$ and z is a point of coincidence of A and S and of B and T with $G_p(z, z, z) = 0$.

We prove that z is the unique point of coincidence of A and S and of B and T .

Suppose that there exists another point of coincidence of A and S , $t = Aw = Sw$. Then, by (4.1) we obtain

$$F\left(\begin{matrix} \psi(G_p(Aw, Bu, Bu)), \psi(G_p(Sw, Tu, Tu)), \psi(G_p(Sw, Sw, Aw)), \\ \psi(G_p(Tu, Bu, Bu)), \psi(G_p(Sw, Bu, Bu)), \psi(G_p(Aw, Tu, Tu)) \end{matrix}\right) \leq 0,$$

$$F\left(\begin{matrix} \psi(G_p(Sw, Tu, Tu)), \psi(G_p(Sw, Tu, Tu)), \psi(G_p(Sw, Sw, Sw)), \\ 0, \psi(G_p(Sw, Tu, Tu)), \psi(G_p(Sw, Tu, Tu)) \end{matrix}\right) \leq 0,$$

$$F\left(\begin{matrix} \psi(G_p(Sw, Tu, Tu)), \psi(G_p(Sw, Tu, Tu)), 0, \\ 0, \psi(G_p(Sw, Tu, Tu)), \psi(G_p(Sw, Tu, Tu)) \end{matrix}\right) \leq 0,$$

which contradicts (F_3) , if $G_p(Sw, Tu, Tu) > 0$. Hence, $G_p(Sw, Tu, Tu) = 0$ which by Lemma 2.2 (1) implies that $Sw = Tu = z$. Hence, $t = z$ and z is the unique point of coincidence of A and S .

Similarly, by (4.1), (F_1) and (F_2) , we obtain that z is the unique point of coincidence of B and T .

Hence, z is the unique point of coincidence of (A, S) and (B, T) .

Moreover, if (A, S) and (B, T) are weakly compatible, by Lemma 4.1, z is the unique common fixed point of A, B, S and T and $G_p(z, z, z) = 0$. \square

If $\psi(t) = t$, we obtain

THEOREM 4.2. Let A, B, S and T be self mappings of a G_p -metric space (X, G_p) satisfying

$$F \left(\begin{array}{l} G_p(Ax, By, By), G_p(Sx, Ty, Ty), G_p(Sx, Sx, Ax), \\ G_p(Ty, By, By), G_p(Sx, By, By), G_p(Ax, Ty, Ty) \end{array} \right) \leq 0$$

for all $x, y \in X$, where $F \in \mathcal{F}_{G_p}$.

If (A, S) and T satisfy $CLR_{(A,S)T}$ -property, then

- 1) $\mathcal{C}(A, S) \neq \emptyset$,
- 2) $\mathcal{C}(B, T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point z and $G_p(z, z, z) = 0$.

EXAMPLE 4.1. Let $X = [0, 1]$ and $G_p(x, y, z) = \max\{x, y, z\}$. Then (X, G_p) is a G_p -metric space.

Consider the following mappings:

$$Ax = 0, \quad Sx = \frac{x}{x+1}, \quad Bx = \frac{x}{3}, \quad Tx = x.$$

Then $S(X) = [0, \frac{1}{2}]$, $T(X) = [0, 1]$ and $S(X) \cap T(X) = [0, \frac{1}{2}]$.

Let $\{x_n\}$ be a sequence in X such that $\lim_{n \rightarrow \infty} x_n = 0$. Then,

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 0 = z \in S(X) \cap T(X).$$

Hence, (A, S) and T satisfy $CLR_{(A,S)T}$ -property with $G_p(0, 0, 0) = 0$.

$Ax = Sx$ implies $\mathcal{C}(A, S) = \{0\}$ and $Bx = Tx$ implies $\mathcal{C}(B, T) = \{0\}$. Moreover, $AS0 = SA0 = 0$ and $BT0 = TB0 = 0$. Hence, (A, S) and (B, T) are weakly compatible. On the other hand,

$$G_p(Ax, By, By) = \frac{y}{3} \quad \text{and} \quad G_p(Ty, By, By) = y.$$

Hence, $G_p(Ax, By, By) \leq ky$, where $k \in [\frac{1}{3}, 1)$, which implies

$$G_p(Ax, By, By) \leq k \max\{G_p(Sx, Ty, Ty), G_p(Sx, Sx, Ax), \\ G_p(Ty, By, By), G_p(Sx, By, By), G_p(Ax, Ty, Ty)\},$$

where $k \in [\frac{1}{3}, 1)$. By Theorem 4.1 and Example 3.2, A, B, S and T have a unique common fixed point $z = 0$ and $G_p(z, z, z) = 0$.

5. Applications

5.1. Fixed points for mappings satisfying contractive conditions of integral type in G_p -metric spaces

In [8], Branciari established the following theorem, which opened the way to the study of fixed points for mappings satisfying a contractive condition of integral type.

THEOREM 5.1 ([8]). *Let (X, d) be a complete metric space, $c \in (0, 1)$ and $f: X \rightarrow X$ be a mapping such that for all $x, y \in X$*

$$\int_0^{d(fx, fy)} h(t) dt \leq c \int_0^{d(x, y)} h(t) dt,$$

where $h: [0, \infty) \rightarrow [0, \infty)$ is a Lebesgue measurable mapping, integrable on each compact subset of $[0, \infty)$, such that $\int_0^\varepsilon h(t)dt > 0$ for $\varepsilon > 0$. Then f has a unique fixed point $z \in X$ and $z = \lim_{n \rightarrow \infty} f^n x$ for all $x \in X$.

Theorem 5.1 has been extended to a pair of compatible mappings in [25].

Some fixed point results for mappings satisfying contractive conditions of integral type have been obtained in [42, 46, 47, 54] and in other papers.

LEMMA 5.1 ([47]). *Let $h: [0, \infty) \rightarrow [0, \infty)$ be as in Theorem 5.1. Then $\psi(x) = \int_0^x h(t)dt$ is an almost altering distance.*

PROOF. It follows by [47, Lemma 2.5]. □

THEOREM 5.2. *Let A, B, S and T be self mappings of a G_p -metric space (X, G_p) such that*

$$(5.1) \quad F \left(\int_0^{G_p(Ax, By, By)} h(t) dt, \int_0^{G_p(Sx, Ty, Ty)} h(t) dt, \int_0^{G_p(Sx, Sx, Ax)} h(t) dt, \int_0^{G_p(Ty, By, By)} h(t) dt, \int_0^{G_p(Sx, By, By)} h(t) dt, \int_0^{G_p(Ax, Ty, Ty)} h(t) dt \right) \leq 0$$

for all $x, y \in X$, where $F \in \mathcal{F}_{G_p}$ and $h(t)$ is as in Theorem 5.1.

If (A, S) and T satisfy $CLR_{(A,S)T}$ -property, then

- 1) $\mathcal{C}(A, S) \neq \emptyset$,
- 2) $\mathcal{C}(B, T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point z and $G_p(z, z, z) = 0$.

PROOF. Taking $\psi(x) = \int_0^x h(t)dt$ we obtain

$$\begin{aligned}\psi(G_p(Ax, By, By)) &= \int_0^{G_p(Ax, By, By)} h(t) dt, \\ \psi(G_p(Sx, Ty, Ty)) &= \int_0^{G_p(Sx, Ty, Ty)} h(t) dt, \\ \psi(G_p(Sx, Sx, Ax)) &= \int_0^{G_p(Sx, Sx, Ax)} h(t) dt, \\ \psi(G_p(Ty, By, By)) &= \int_0^{G_p(Ty, By, By)} h(t) dt, \\ \psi(G_p(Sx, By, By)) &= \int_0^{G_p(Sx, By, By)} h(t) dt, \\ \psi(G_p(Ax, Ty, Ty)) &= \int_0^{G_p(Ax, Ty, Ty)} h(t) dt.\end{aligned}$$

Then, by (5.1) we obtain

$$F\left(\psi(G_p(Ax, By, By)), \psi(G_p(Sx, Ty, Ty)), \psi(G_p(Sx, Sx, Ax)), \right. \\ \left. \psi(G_p(Ty, By, By)), \psi(G_p(Sx, By, By)), \psi(G_p(Ax, Ty, Ty))\right) \leq 0,$$

that is inequality (4.1). Moreover, by Lemma 5.1, ψ is an almost altering distance.

Hence, the conditions of Theorem 4.1 are satisfied and the conclusions of Theorem 5.2 follows by Theorem 4.1. \square

By Theorem 5.2 and Example 3.2 we obtain the following.

THEOREM 5.3. *Let A, B, S and T be self mappings of a G_p -metric space (X, G_p) such that*

$$\int_0^{G_p(Ax, By, By)} h(t) dt \leq k \max \left\{ \int_0^{G_p(Sx, Ty, Ty)} h(t) dt, \int_0^{G_p(Sx, Sx, Ax)} h(t) dt, \right. \\ \left. \int_0^{G_p(Ty, By, By)} h(t) dt, \int_0^{G_p(Sx, By, By)} h(t) dt, \int_0^{G_p(Ax, Ty, Ty)} h(t) dt \right\}$$

for all $x, y \in X$, where $k \in [0, 1)$ and $h(t)$ is as in Theorem 5.1.

If (A, S) and T satisfy $CLR_{(A,S)T}$ -property, then

- 1) $\mathcal{C}(A, S) \neq \emptyset$,
- 2) $\mathcal{C}(B, T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point z and $G_p(z, z, z) = 0$.

EXAMPLE 5.1. Let $X = [0, \infty)$ and $G_p(x, y, z) = \max\{x, y, z\}$. Then (X, G_p) is a G_p -metric space.

Consider the following mappings:

$$Ax = \frac{x}{2}, \quad Sx = 2x, \quad Bx = 0, \quad Tx = x.$$

Then $S(X) = [0, \infty), T(X) = [0, \infty)$ and $S(X) \cap T(X) = [0, \infty)$.

Let $\{x_n\}$ be a sequence in X such that $\lim_{n \rightarrow \infty} x_n = 0$. Then

$$\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ax_n = 0 = z \in S(X) \cap T(X)$$

and $G_p(z, z, z) = G(0, 0, 0) = 0$. Hence, (A, S) and T satisfy $CLR_{(A,S)T}$ -property.

$Ax = Sx$ implies $\mathcal{C}(A, S) = \{0\}$ and $Bx = Tx$ implies $\mathcal{C}(B, T) = \{0\}$. Moreover, $AS0 = SA0 = 0$ and $BT0 = TB0 = 0$. Hence, (A, S) and (B, T) are weakly compatible. On the other hand,

$$G_p(Ax, By, By) = Ax = \frac{x}{2} \quad \text{and} \quad G_p(Sx, Sx, Ax) = 2x.$$

Moreover,

$$\int_0^{x/2} t \, dt \leq k \int_0^{2x} t \, dt$$

for $k \geq \frac{1}{16}$. Thus, for $h(t) = t$ we obtain

$$\int_0^{G_p(Ax, By, By)} h(t) \, dt \leq k \int_0^{G_p(Sx, Sx, Ax)} h(t) \, dt,$$

where $\frac{1}{16} \leq k < 1$. Hence,

$$\int_0^{G_p(Ax, By, By)} h(t) \, dt \leq k \max \left\{ \int_0^{G_p(Sx, Ty, Ty)} h(t) \, dt, \int_0^{G_p(Sx, Sx, Ax)} h(t) \, dt, \int_0^{G_p(Ty, By, By)} h(t) \, dt, \int_0^{G_p(Sx, By, By)} h(t) \, dt, \int_0^{G_p(Ax, Ty, Ty)} h(t) \, dt \right\},$$

where $k \in [\frac{1}{16}, 1)$. By Theorem 5.3, A, B, S and T have a unique common fixed point $z = 0$.

REMARK 5.1. By Theorem 5.2 and Examples 3.1, 3.3–3.8 we obtain new particular results.

5.2. Fixed points for mappings satisfying φ -contractive conditions in G_p -metric spaces

As in [27], let Φ be the set of real continuous nondecreasing functions $\varphi: [0, \infty) \rightarrow [0, \infty)$ with $\lim_{n \rightarrow \infty} \varphi^n(t) = 0$.

If $\varphi \in \Phi$, then

- (1) $\varphi(t) < t$ for all $t \in (0, \infty)$,
- (2) $\varphi(0) = 0$.

The following functions $F: \mathbb{R}_+^6 \rightarrow \mathbb{R}_+$ satisfy conditions (F_1) , (F_2) , (F_3) .

EXAMPLE 5.2. $F(t_1, \dots, t_6) = t_1 - \varphi(\max\{t_2, t_3, \dots, t_6\})$.

EXAMPLE 5.3. $F(t_1, \dots, t_6) = t_1 - \varphi(\max\{t_2, t_3, t_4, \frac{t_5+t_6}{2}\})$.

EXAMPLE 5.4. $F(t_1, \dots, t_6) = t_1 - \varphi(\max\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\})$.

EXAMPLE 5.5. $F(t_1, \dots, t_6) = t_1 - \varphi(\max\{t_2, t_3, t_4, \sqrt{t_5 t_6}\})$.

EXAMPLE 5.6. $F(t_1, \dots, t_6) = t_1 - \varphi(at_2 + bt_3 + ct_4 + dt_5 + et_6)$, where $a, b, c, d, e \geq 0$ and $a + b + c + d + e < 1$.

EXAMPLE 5.7. $F(t_1, \dots, t_6) = t_1 - \varphi(at_2 + b \max\{t_3, t_4\} + c \max\{\frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\})$, where $a, b, c \geq 0$ and $a + b + c < 1$.

By Theorem 4.1 and Example 5.3 we obtain

THEOREM 5.4. *Let A, B, S and T be self mappings of a G_p -metric space (X, G_p) such that*

$$\psi(G_p(Ax, By, By)) \leq \varphi\left(\max\left\{\psi(G_p(Sx, Ty, Ty)), \psi(G_p(Sx, Sx, Ax)), \psi(G_p(Ty, By, By)), \frac{\psi(G_p(Sx, By, By)) + \psi(G_p(Ax, Ty, Ty))}{2}\right\}\right)$$

for all $x, y \in X$, where $\varphi \in \Phi$ and ψ is an almost altering distance.

If (A, S) and T satisfy $CLR_{(A,S)T}$ -property, then

- 1) $\mathcal{C}(A, S) \neq \emptyset$,
- 2) $\mathcal{C}(B, T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point z and $G_p(z, z, z) = 0$.

EXAMPLE 5.8. Let (X, G_p) , A, S, B and T be as in Example 5.1. Put $\varphi(t) = \frac{t}{2}$. Obviously, $\varphi \in \Phi$.

It follows from Example 5.1 that (A, S) and T satisfy $CLR_{(A,S)T}$ -property, (A, S) and (B, T) are weakly compatible and

$$G_p(Ax, By, By) = Ax = \frac{x}{2} \text{ and } G_p(Sx, Sx, Ax) = 2x,$$

which implies

$$\begin{aligned} G_p(Ax, By, By) &\leq \frac{1}{2}G_p(Sx, Sx, Ax) \\ &\leq \frac{1}{2} \max\{G_p(Sx, Ty, Ty), G_p(Sx, Sx, Ax), \\ &\quad G_p(Ty, By, By), \frac{G_p(Sx, By, By) + G_p(Ax, Ty, Ty)}{2}\} \\ &= \varphi\left(\max\{G_p(Sx, Ty, Ty), G_p(Sx, Sx, Ax), \right. \\ &\quad \left. G_p(Ty, By, By), \frac{G_p(Sx, By, By) + G_p(Ax, Ty, Ty)}{2}\right). \end{aligned}$$

By Theorem 5.4, A, B, S and T have a unique common fixed point $z = 0$.

By Theorem 5.2 and Example 5.3 we obtain the following.

THEOREM 5.5. Let A, B, S and T be self mappings of a G_p -metric space (X, G_p) such that

$$\int_0^{G_p(Ax, By, By)} h(t) dt \leq \varphi\left(\max\left\{\int_0^{G_p(Sx, Ty, Ty)} h(t) dt, \int_0^{G_p(Sx, Sx, Ax)} h(t) dt, \int_0^{G_p(Ty, By, By)} h(t) dt, \frac{\int_0^{G_p(Sx, By, By)} h(t) dt + \int_0^{G_p(Ax, Ty, Ty)} h(t) dt}{2}\right\}\right)$$

for all $x, y \in X$, where $h(t)$ is as in Theorem 5.1.

If (A, S) and T satisfy $CLR_{(A,S)T}$ -property, then

- 1) $C(A, S) \neq \emptyset$,
- 2) $C(B, T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point z and $G_p(z, z, z) = 0$.

REMARK 5.2. By Theorem 5.2 and Examples 5.2, 5.4–5.7 we obtain new particular results.

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