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ALTERING DISTANCE AND COMMON FIXED POINTS FOR HYBRID MAPPINGS UNDER IMPLICIT RELATIONS AND APPLICATIONS

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Abstract. In this paper we prove a general fixed point theorem by altering distance for two pairs of owc hybrid mappings generalizing the main result from [10] and we reduce the study of fixed point of pairs of mappings satisfying a contractive condition of integral type at the study of fixed points in metric spaces by altering distance satisfying an implicit relation.

1. Introduction and Preliminaries

Let (X,d) be a metric space and let $B(X)$ be the set of all nonempty bounded subsets of X. As in [15], [16] we define the functions $D(A,B)$ and $\delta(A,B)$, where $A,B \in B(X)$ by

$$D(A,B) = \inf\{d(a,b) : a \in A, b \in B\},$$

$$\delta(A,B) = \sup\{d(a,b) : a \in A, b \in B\}.$$

If A consists of a single point a, we write $\delta(A,B) = \delta(a,B)$. If B consists also of a single point b, we write $\delta(A,B) = d(a,b)$.

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It follows immediately from definition of δ that

$$\delta(A,B) = \delta(B,A), A, B \in B(X).$$

If $\delta(A,B) = 0$ then $A = B = \{a\}$.

Definition 1.1. Let $f\colon X \to X$ and $F\colon X \to B(X)$.

1) A point $x \in X$ is said to be a *coincidence point* of f and F if $fx \in Fx$. We denote by $C(f,F)$ the set of all coincidence points of f and F.
2) A point $x \in X$ is said to be a *fixed point* of F if $x \in Fx$.

Definition 1.2 ([15], [16]). Let (X,d) be a metric space. A sequence $\{A_n\}$ of a nonempty subset of X is said to be convergent to a set A of X if

(i) each point $a \in A$ is the limit of a convergent sequence $\{a_n\}$, where $a_n \in A_n$ for all $n \in \mathbb{N}$,

(ii) for arbitrary $\varepsilon > 0$, there exists an integer $m > 0$ such that $A_n \subset A_\varepsilon$ for $n > m$, where A_ε is the set of all points $x \in X$ for which there exists a point $a \in X$, depending on x, such that $d(x,a) < \varepsilon$.

A is said to be the limit of the sequence $\{A_n\}$.

Definition 1.3 ([18]). The mappings $f\colon X \to X$ and $F\colon X \to B(X)$ are *δ-compatible* if $\lim_{n\to\infty} \delta(Ffx_n, fFx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $fFx_n \in B(X)$, $fx_n \to t$, $Fx_n \to \{t\}$ for some $t \in X$.

Definition 1.4 ([19]). The pair $f\colon X \to X$ and $F\colon X \to B(X)$ is *weakly compatible* if for each $x \in C(f,F)$, $fFx = Ffx$.

If the pair (f,F) is δ-compatible, then (f,F) is weakly compatible but the converse is not true [19].

The notions of occasionally weakly compatible single valued functions is introduced in [8]. Some fixed points theorems for occasionally weakly compatible single valued functions are proved in [20]. There exists a vast literature of this topic.

The notion of occasionally weakly compatible hybrid mappings is first introduced in [1].

Definition 1.5. The hybrid pair $f\colon X \to X$ are $F\colon X \to B(X)$ is *occasionally weakly compatible* (owc) if there exists $x \in C(f,F)$ such that $fFx \subset Ffx$.

Remark 1.1. Every weakly compatible pair of mappings is owc. The converse is not true (Example 1.3 [1]).

Theorem 1.1 ([10]). *Let $f, g\colon X \to X$ be the maps and $F, G\colon X \to B(X)$ be set valued maps such that $\{f, F\}$ and $\{g, G\}$ are owc. Let $\varphi\colon \mathbb{R}^+ \to \mathbb{R}^+$ be a nondecreasing map such that for every $t > 0$, $\varphi(t) < t$, satisfying the following condition:*

$$\delta^p(Fx, Gy) < \varphi(ad(fx, gy) + (1-a)\max\{\alpha D^p(fx, Fx), \beta D^p(gy, Gy),$$
$$[D(fx, Fx)\cdot D(gy, Fx)]^{p/2}, [D(gy, Fx)\cdot D(fx, Gy)]^{p/2},$$
$$\frac{1}{2}[D^p(fx, Gy) + D^p(gy, Fx)]\})$$

for all $x, y \in X$, where $0 < a \leq 1$, $0 < \alpha, \beta < 1$ and $p \geq 1$.

Then, f, g, F and G have a unique common fixed point.

Definition 1.6. An *altering distance* is a mapping $\psi\colon [0,\infty) \to [0,\infty)$ which satisfies the following conditions:
(i) ψ is increasing and continuous,
(ii) $\psi(t) = 0$ if and only if $t = 0$.

Fixed point problem involving an altering distance have been studied in [21], [27], [29], [30] and in other papers.

In [25] a general fixed point theorem for compatible mappings satisfying an implicit relation is proved. In [17] the results from [25] are improved relaxing the compatibility to weak compatibility.

2. Implicit relations

Definition 2.1. Let $\mathfrak{F}_W$ be the set of all functions $\phi(t_1, \ldots, t_6) : \mathbb{R}^6_+ \to \mathbb{R}$ satisfying the following conditions:

(ϕ_1): ϕ is nondecreasing in variable t_1 and nonincreasing in variables t_5 and t_6,
(ϕ_2): $\phi(t, t, 0, 0, t, t) \geq 0$, $\forall t > 0$.

Example 2.1. $\phi(t_1, \ldots, t_6) = t_1 - \varphi\max\{t_2, \ldots, t_6\}$, *where* $\varphi\colon \mathbb{R}^+ \to \mathbb{R}^+$ *with* $\varphi(t) < t$, $\forall t > 0$.

Example 2.2. $\phi(t_1, \ldots, t_6) = t_1^p - \varphi(at_2^p + (1-a)\max\{\alpha t_3^p, \beta t_4^p, (t_3t_6)^{p/2}, (t_5t_6)^{p/2}, \frac{1}{2}(t_3^p + t_4^p)\})$, *where* $0 < a < 1, 0 < \alpha, \beta < 1$ *and* $p \geq 1$, $\varphi\colon \mathbb{R}^+ \to \mathbb{R}^+$ *with* $\varphi(t) < t$, $\forall t > 0$.

Example 2.3. $\phi(t_1,\ldots,t_6) = t_1 - \varphi\left(\max\left\{t_2,t_3,t_4,\frac{1}{2}(t_5+t_6)\right\}\right)$, *where* $\varphi\colon \mathbb{R}^+ \to \mathbb{R}^+$ *with* $\varphi(t) < t$, $\forall t > 0$.

Example 2.4. $\phi(t_1,\ldots,t_6) = t_1 - \alpha \max\{t_2,t_3,t_4\} - (1-\alpha)(at_5+bt_6)$, *where* $0 < \alpha < 1, a, b \geq 0$ *and* $a+b<1$.

Example 2.5. $\phi(t_1,\ldots,t_6) = t_1 - at_2 - b(t_3+t_4) - c(t_5+t_6)$, *where* $a > 0$, $b, c \geq 0$ *and* $a + 2c \leq 1$.

Example 2.6. $\phi(t_1,\ldots,t_6) = t_1 - \max\left\{t_2, \frac{1}{2}(t_3+t_4), \frac{1}{2}(t_5+t_6)\right\}$.

Example 2.7. $\phi(t_1,\ldots,t_6) = t_1 - \max\{ct_2, ct_3, ct_4, at_5+bt_6\}$, *where* $0 < c \leq 1$, $a \geq 0$, $b \geq 0$ *and* $a+b<1$.

For other examples see [4].

In [10], the following theorem is proved.

Theorem 2.1. *Let* (X,d) *be a symmetric space and let* $f, g\colon X \to X$ *and* $F\colon X \to B(X)$ *be the set valued maps such that the pairs* $\{f,F\}$ *and* $\{g,G\}$ *are owc. Let* $\varphi\colon \mathbb{R}^6_+ \to \mathbb{R}$ *be a real map satisfying the following conditions:*

(φ_1)*:* φ *is nonincreasing in variable* t_1 *and nonincreasing in variables* t_5 *and* t_6*,*

(φ_2)*:* $\varphi(t,t,0,0,t,t) \geq 0$, $\forall t > 0$.

If

$$\varphi(\delta(Fx,Gy), d(fx,gy), D(fx,Fx), D(gy,Gy), D(fx,Gy), D(gy,Fx)) < 0$$

for all $x, y \in X$ *for which*

$$\max\{d(fx,gy), D(fx,Fx), D(gy,Gy)\} > 0,$$

then, f, g, F *and* G *have a unique common fixed point.*

The purpose of this paper is to prove a general fixed point theorem by altering distance for two pairs of owc hybrid mappings generalizing Theorem 2.1 and to reduce the study of fixed point of pairs of mappings satisfying a contractive condition of integral type at the study of fixed points in metric spaces by altering distance satisfying an implicit relation.

3. General fixed point theorem for owc hybrid mappings

Theorem 3.1. *Let $f, g\colon X \to X$ be maps and $F\colon X \to B(X)$ be set-valued maps such that the pairs $\{f, F\}$ and $\{g, G\}$ are owc. If*

$$(3.1)\quad \phi(\psi(\delta(Fx, Gy)), \psi(d(fx, gy)), \psi(D(fx, Fx)), \psi(D(gy, Gy)), \psi(D(fx, Gy)), \psi(D(gy, Fx))) < 0$$

for all $x, y \in X$, with $fx \neq gy$, $\phi \in \mathfrak{F}_W$ and ψ is an altering distance, then f, g, F and G have a unique common fixed point.

Proof. Since the pairs $\{f, F\}$ and $\{g, G\}$ are owc then, there exist u, v in X such that $fu \in Fu$, $gv \in Gv$, $fFu \subset Ffu$, $gGv \subset Ggv$. First we show that $fu = gv$. Suppose that $fu \neq gv$. Then by (3.1) we have successively

$$\phi(\psi(\delta(Fu, Gv)), \psi(d(fu, gv)), \psi(D(fu, Fu)), \psi(D(gv, Gv)), \psi(D(fu, Gv), \psi(D(gv, Fu))) < 0,$$

$$\phi(\psi(\delta(Fu, Gv)), \psi(d(fu, gv)), 0, 0, \psi(D(fu, Gv), \psi(D(gv, Fu))) < 0.$$

By (ϕ_1) we have

$$\phi(\psi(d(fu, gv)), \psi(d(fu, gv)), 0, 0, \psi(d(fu, gv), \psi(d(fu, gv))) < 0,$$

a contradiction of (ϕ_2), which implies $fu = gv$.

Next we claim that $fu = f^2u$. If $fu \neq f^2u$, by (3.1) we have

$$\phi(\psi(\delta(Ffu, Gv)), \psi(d(f^2u, gv)), \psi(D(f^2u, Ffu)), \psi(D(gv, Gv)), \psi(D(f^2u, Gv), \psi(D(gv, Ffu))) < 0,$$

$$\phi(\psi(\delta(Ffu, Gv)), \psi(d(f^2u, gv)), 0, 0, \psi(D(f^2u, Gv), \psi(D(gv, Ffu))) < 0.$$

By (ϕ_1) we have

$$\phi(\psi(d(f^2u, fu)), \psi(d(f^2u, fu)), 0, 0, \psi(d(f^2u, fu), \psi(d(f^2u, fu))) < 0,$$

a contradiction of (ϕ_2). Hence, $fu = f^2u$. Similarly, $gv = g^2v$. Therefore, $ffu = fu = gv = g^2v = gfu$. Hence fu is a common fixed point of f and g. On the other hand we have $fu = f^2u \in fFu \subset Ffu$. Hence fu is a fixed point of F. Similarly, $fu = gv = g^2v \in gGv \subset Ggv = Gfu$ and fu is a fixed point of G. Therefore, f, g, F and G have a common fixed point $w = fu$.

Suppose that $w' \neq w$ is another common fixed point of f, g, F and G. By (3.1) we obtain

$$\phi(\psi(\delta(Fw, Gw')), \psi(d(fw, gw')), \psi(D(fw, Fw)),$$
$$\psi(D(gw', Gw')), \psi(D(fw, Gw'), \psi(D(fw, Gw'))) < 0.$$

By (ϕ_1) we obtain

$$\phi(\psi(d(fw, gw')), \psi(d(fw, gw')), 0, 0, \psi(d(fw, gw'), \psi(d(gw', fw))) < 0,$$
$$\phi(\psi(d(w, w')), \psi(d(w, w')), 0, 0, \psi(d(w, w'), \psi(d(w', w))) < 0,$$

a contradiction of (ϕ_2). □

Remark 3.1. 1) If $\psi(t) = t$ we obtain Theorem 2.1.
2) By Theorem 3.1, $\psi(t) = t$ and Example 2.2 we obtain Theorem 1.1.
3) Theorem 3.1 generalizes Theorems from [2], [5], [9], [14], [15], [31] and other papers.

Let $\mathfrak{F}_W^*$ be the set of all self functions $\phi(t_1, \ldots, t_6)\colon \mathbb{R}_+^6 \to \mathbb{R}$ satisfying condition (ϕ_2).

If f, g, F and G are single valued functions we have

Theorem 3.2. *Let $f, g, F, G\colon X \to X$ be self maps on a metric space (X, d) such that $\{f, F\}$ and $\{g, G\}$ are owc. If*

$$\phi(\psi(d(Fx, Gy)), \psi(d(fx, gy)), \psi(d(fx, Fx)),$$
$$\psi(d(gy, Gy)), \psi(d(fx, Gy)), \psi(d(gy, Fx))) < 0 \tag{3.2}$$

for all $x, y \in X$, with $fx \neq gy$, $\phi \in \mathfrak{F}_W^*$ and ψ is an altering distance, then, f, g, F and G have a unique common fixed point.

Proof. The proof is similar with the proof of Theorem 3.1. □

If $\psi(t) = t$, then by Theorem 3.2 we obtain

THEOREM 3.3. *Let* $f, g, F, G\colon X \to X$ *be self maps on a metric space* (X, d) *such that* $\{f, F\}$ *and* $\{g, G\}$ *are owc. If*

$$\phi(d(Fx, Gy), d(fx, gy), d(fx, Fx), d(gy, Gy), d(fx, Gy), d(gy, Fx)) < 0 \tag{3.3}$$

for all $x, y \in X$, *with* $fx \neq gy$ *and* $\phi \in \mathfrak{F}^*_W$, *then,* f, g, F *and* G *have a unique common fixed point.*

REMARK 3.2. By Examples 2.1–2.7 and other examples from [4] and Theorem 3.1 we get the Theorems from [20] and several known results and new results.

4. Altering distance and common fixed points for hybrid pairs satisfying a contractive condition of integral type

In [11], Branciari established the following result

THEOREM 4.1. *Let* (X, d) *be a complete metric space,* $c \in (0, 1)$ *and* $f\colon X \to X$ *be a mapping such that*

$$\int_0^{d(fx,fy)} h(t)dt \leq c \int_0^{d(x,y)} h(t)dt \tag{4.1}$$

where $h\colon [0, \infty) \to [0, \infty)$ *is a Lebesgue measurable mapping which is summable (i.e. with a finite integral) on each compact subset of* $[0, \infty)$ *such that for* $\varepsilon > 0$, $\int_0^{\varepsilon} h(t)dt > 0$. *Then,* f *has a unique fixed point* $z \in X$ *such that for each* $x \in X$, $\lim_{n\to\infty} f^n x = z$.

Some fixed point theorems in metric and symmetric spaces for compatible, weak compatible and occasionally weakly compatible mappings satisfying a contractive condition of integral type are proved in [3], [22], [23], [24], [26], [27], [28] and in other papers.

LEMMA 4.1. *The function* $\psi(x) = \int_0^x h(t)dt$, *where* $h(t)$ *is as in Theorem 4.1, is an altering distance.*

PROOF. From the definitions of h and ψ it follows that $\psi(t)$ is increasing and $\psi(x) = 0$ if and only if $x = 0$. Obviously, $\psi(x)$ is continuous. □

Theorem 4.2. *Let f and g be self maps of a metric space (X, d) and F, G be maps of X into $B(X)$ such that $\{f, F\}$ and $\{g, G\}$ are owc. If*

$$(4.2)\quad \phi\Big(\int_0^{\delta(Fx,Gy)} h(t)dt, \int_0^{d(fx,gy)} h(t)dt, \int_0^{D(fx,Fx)} h(t)dt, \int_0^{D(gy,Gy)} h(t)dt, \int_0^{D(fx,Gy)} h(t)dt, \int_0^{D(gy,Fx)} h(t)dt\Big) < 0$$

for all $x, y \in X$, with $fx \neq gy$, $\phi \in \mathfrak{F}_W$ and $h(t)$ is as in Theorem 4.1, then, f, g, F and G have a unique common fixed point.

Proof. As in Lemma 4.1 we have

$$\psi(\delta(Fx, Gy)) = \int_0^{\delta(Fx,Gy)} h(t)dt, \psi(d(fx, gy)) = \int_0^{d(fx,gy)} h(t)dt,$$

$$\psi(D(fx, Fx)) = \int_0^{D(fx,Fx)} h(t)dt, \psi(D(gy, Gy)) = \int_0^{D(gy,Gy)} h(t)dt,$$

$$\psi(D(fx, Gy)) = \int_0^{D(fx,Gy)} h(t)dt, \psi(D(gy, Fx)) = \int_0^{D(gy,Fx)} h(t)dt.$$

Then by (4.2) we obtain

$$\phi(\psi(\delta(Fx, Gy)), \psi(d(fx, gy)), \psi(D(fx, Fx)), \psi(D(gy, Gy)), \psi(D(fx, Gy)), \psi(D(gy, Fx))) < 0$$

for all $x, y \in X$, with $fx \neq gy$ and $\phi \in \mathfrak{F}_W$, which is the inequality (3.1). Because by Lemma 4.1, $\psi(t) = \int_0^t h(x)dx$ is an altering distance, then the conditions of Theorem 3.1 are satisfied and Theorem 4.2 follows from Theorem 3.1. □

Remark 4.1. If $h(t) = 1$ then by Theorem 4.2 we obtain the results from Theorem 2.1.

If f, g, F and G are single valued functions we have

THEOREM 4.3. *Let f, g, F and G be single valued self maps of a metric space (X, d) such that $\{f, F\}$ and $\{g, G\}$ are owc. If*

$$(4.3)\quad \phi\Big(\int_0^{d(Fx,Gy)} h(t)dt, \int_0^{d(fx,gy)} h(t)dt, \int_0^{d(fx,Fx)} h(t)dt, \int_0^{d(gy,Gy)} h(t)dt, \int_0^{d(fx,Gy)} h(t)dt, \int_0^{d(gy,Fx)} h(t)dt\Big) < 0$$

for all $x, y \in X$ for which $fx \neq gy$, where $\phi \in \mathfrak{F}_W^*$ and $h(t)$ is as in Theorem 4.1, then, f, g, F and G have a unique common fixed point.

PROOF. The proof is similar with the proof of Theorem 4.2 and follows from Theorem 3.2. □

REMARK 4.2. By Theorems 4.2, 4.3 and Examples 2.1–2.4 we get generalizations of Theorems from [3], [7], [13], Theorem 2.1 [2], Theorems from [12] and Theorems from [6]. New results we obtain from Examples 2.5–2.7 and other examples from [4].

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