Report of Meeting

The Fourteenth Debrecen–Katowice Winter Seminar
Hajdúszoboszló (Hungary),
January 29 – February 1, 2014

The Fourteenth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities was held in Hotel Aurum, Hajdúszoboszló, Hungary, from January 29 to February 1, 2014. It was organized by the Department of Analysis of the Institute of Mathematics of the University of Debrecen.

The Winter Seminar was supported by the following organizations:

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27 participants came from the Silesian University of Katowice (Poland) and the University of Debrecen (Hungary), 13 from the former and 14 from the latter city.

Professor Zsolt Páles opened the Seminar and welcomed the participants to Hajdúszoboszló.

The scientific talks presented at the Seminar focused on the following topics: equations in a single variable and in several variables, iterative equations, equations on algebraic structures, regularity properties of the solutions of certain functional equations, functional inequalities, Hyers–Ulam stability, functional equations and inequalities involving mean values, generalized convexity. Interesting discussions were generated by the talks.

There were profitable Problem Sessions.

The social program included a Festive Dinner. Furthermore, the participants had the opportunity to take advantage of the use of the thermal bath located in the hotel.

The closing address was given by Professor Roman Ger. His invitation to the Fifteenth Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities in January 2015 in Poland was gratefully accepted.

Summaries of the talks in alphabetic order of the authors follow in section 1, problems and remarks in chronological order in section 2, and the list of participants in the final section.

1. Abstracts of talks

ROMAN BADORA: *Functional inequalities in lattices* (Joint work with Tomasz Kochanek and Barbara Przebieracz)

We continue the research on the functional inequalities in lattices.

MIHÁLY BESSENYEI: *A contraction principle in semimetric spaces* (Joint work with Zsolt Páles)

A branch of generalizations of the Banach Fixed Point Theorem replaces contractivity by a weaker but still effective property. The aim of the present talk is to extend the contraction principle in this spirit for such complete semimetric spaces that fulfill an extra regularity property. The stability of fixed points is also investigated in this setting.

ZOLTÁN BOROS: *A regularity condition for quadratic functions involving the unit circle* (Joint work with Włodzimierz Fechner and Péter Kutas)

Kominek, Reich and Schwaiger [KRS] proved the following theorem: If $f: \mathbb{R} \to \mathbb{R}$ is additive and $f(x)f(y) = 0$ for all solutions of the equation $x^2 + y^2 = 1$, then f is identically equal to zero. The author together with the first coauthor [BF] extended this result to the case when f is a a generalized polynomial function. In this talk we investigate the stability of the condition $f(x)f(y) = 0$ on the unit circle by replacing it with the assumption that $f(x)f(y)$ is bounded there. We establish the following results:

THEOREM 1. *If $f: \mathbb{R} \to \mathbb{R}$ is additive or quadratic, respectively, and there exists a non-void open subinterval $I \subset]0,1[$ such that the mapping*

$$\phi(x) = f(x)f(\sqrt{1 - x^2})$$

is bounded on I, then $f(x) = cx^m$ ($x \in \mathbb{R}$) with some real coefficient c and $m = 1$ or $m = 2$, respectively.

THEOREM 2. *If $f\colon \mathbb{R} \to \mathbb{R}$ is a generalized polynomial of degree (at most)* 2, $f(0) = 0$, *and the mapping $\Psi(x,y) = f(x)f(y)$ is bounded on the set*

$$S = \{\, (x,y) \in \mathbb{R}^2 \,|\, x^2 + y^2 = 1 \,\},$$

then f is continuous.

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BURAI PÁL: *A functional equation from optimization*

Let X be a Banach space. We investigate the following functional equation:

$$\Phi(\Pi(\bar{x}, x, \cdot)) = \tilde{\Pi}(\Phi\bar{x}, \Phi x, \cdot), \qquad \bar{x}, x \in X,$$

where Φ is a self-bijection of X, and the maps $\Pi, \tilde{\Pi}$ constitute generalized convexity structures. Some open problems in connection with this equation are also presented.

SZYMON DRAGA: *On weakly locally uniformly rotund norms which are not locally uniformly rotund*

During the talk we will recall some notions connected with geometry of Banach spaces and Markushevich bases. Following an idea of D. Yost (cf. [2]) we will show that every infinite-dimensional Banach space with separable dual admits an equivalent norm which is weakly locally uniformly rotund but not locally uniformly rotund. The material to be presented is based mainly on the author's master's thesis [1].

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WŁODZIMIERZ FECHNER: *Inequalities related to Hosszú's functional equation*

Hosszú's functional equation is the following equation:

$$(1) \qquad\qquad f(x + y - xy) + f(xy) = f(x) + f(y),$$

where $x, y \in (0, 1)$. Gy. Maksa and Zs. Páles [1] and later J.E. Pečarić [2] and Z. Powązka [3] have dealt with the following two functional inequalities:

$$(2) \qquad f(x + y - xy) \leq f(x) + f(y)$$

and

$$(3) \qquad f(x + y - xy) + f(xy) \leq f(x) + f(y)$$

for function f defined on the open interval $(0, 1)$. They established some connections of solutions of (2) and (3) with Jensen-concave functions and Wright-concave functions.

Answering a question posed by J.M. Rassias (personal communication) we deal with another functional inequality, which is related to (2) and (3):

$$f(x + y + xy) \leq f(x) + f(y) + f(xy).$$

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ROMAN GER: *Mean values for vector valued functions and corresponding functional equations* (Joint work with Maciej Sablik)

It is well known that mean value theorems offered by the classical one-dimensional analysis do not carry over to vector valued mappings. Nevertheless, some substitutes are known like, for instance, Sanderson's and McLeod's results (see [3] and [2], respectively). The latter one, in two-dimensional case, may be formulated in the following way:

THEOREM. *Let $f \colon \mathbb{R} \longrightarrow \mathbb{R}^2$ be continuously differentiable. Then there exist two means $m_i \colon \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$, $i = 1, 2$ and a function $\lambda \colon \mathbb{R} \times \mathbb{R} \longrightarrow [0, 1]$ such that*

$$(*) \quad f(y) - f(x) = (y - x)\left[\lambda(x, y)f'(m_1(x, y)) + (1 - \lambda(x, y))f'(m_2(x, y))\right]$$

for all $x, y \in \mathbb{R}$.

We shall study equation $(*)$ with the coefficient function $\lambda(x,y) \equiv \frac{1}{2}$ and, to get rid of the differentiability assumption, with the derivatives $\frac{1}{2}f'$ in $(*)$ replaced by another unknown function $g \colon \mathbb{R} \longrightarrow \mathbb{R}^2$. That is, we shall examine a Pexider type functional equation

$$\frac{f(y) - f(x)}{y - x} = g(m_1(x,y)) + g(m_2(x,y))$$

with some means m_1 and m_2. Keeping in mind the celebrated Aczél's result characterizing quadratic polynomials (see [1]) we are expecting to have quadratic "polynomials" $f(x) = ax^2 + bx + c$, $x \in \mathbb{R}$, with some fixed vectors $a, b, c \in \mathbb{R}^2$, as potential solutions. This forces the existence of a mean m such that

$$m_1(x,y) = m(x,y) \quad \text{and} \quad m_2(x,y) = x + y - m(x,y).$$

Numerous results of that kind will be discussed and reported on.

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ATTILA GILÁNYI: *On $(t_1, \ldots, t_n)$-Wright-convex functions with a modulus* (Joint work with Nelson Merentes, Kazimierz Nikodem and Zsolt Páles)

During the 13$^{\text{th}}$ Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities, in the talk [1], results on strongly Wright-convex functions of higher order were presented (cf. also [3]). Related to this talk and to Remark [2], we investigate $(t_1, \ldots, t_n)$-Wright-convex functions with a modulus c, i.e., functions $f \colon I \to \mathbb{R}$ satisfying the inequality

$$\Delta_{t_1 h} \cdots \Delta_{t_n h} f(x) \geq cn!(t_1 h) \cdots (t_n h)$$

for all $x \in I$, $h > 0$ such that $x + (t_1 + \cdots + t_n)h \in I$, where n is a positive integer, c is an arbitrary and $t_1, \ldots, t_n$ are positive real numbers and $I \subseteq \mathbb{R}$ is an open interval.

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ESZTER GSELMANN: *Approximate derivations of order n*

For any function $f \colon \mathbb{R} \to \mathbb{R}$ and $\alpha \in \mathbb{R}$ we define

$$\delta_\alpha f(x) = f(\alpha x) - \alpha f(x) \qquad (x \in \mathbb{R}).$$

Clearly, if $f \colon \mathbb{R} \to \mathbb{R}$ is an additive function then

$$\delta_\alpha f(x) = 0 \ (\alpha, x \in \mathbb{R}) \quad \text{or} \quad \delta_\alpha f(1) = 0 \ (\alpha \in \mathbb{R})$$

yield that f is linear.

Let $n \in \mathbb{N}$ be fixed. Following the work of J. Unger and L. Reich (*Derivationen höherer Ordnung als Lösungen von Funktionalgleichungen*, volume 336 of *Grazer Mathematische Berichte [Graz Mathematical Reports]*, Karl-Franzens-Universität Graz, Graz, 1998.), an additive function $f \colon \mathbb{R} \to \mathbb{R}$ is said to be a *derivation of order n* if,

$$f(1) = 0 \quad \text{and} \quad \delta_{\alpha_1} \circ \cdots \circ \delta_{\alpha_{n+1}} f(x) = 0$$

is fulfilled for any $x, \alpha_1, \ldots, \alpha_{n+1} \in \mathbb{R}$.

From this notion immediately follows that first order derivations are just real derivations. If we drop the assumption $f(1) = 0$, we get the $f \colon \mathbb{R} \to \mathbb{R}$ fulfills

$$\delta_\alpha \delta_\beta f(x) = 0 \qquad (\alpha, \beta, x \in \mathbb{R})$$

if and only if

$$f(x) = d(x) + f(1)x,$$

where d is a real derivation.

The aim of this talk is to prove characterization theorems for higher order derivations. Among others we prove that the system defining higher order

derivations is stable. Further characterization theorems in the spirit of N.G. de Bruijn will also be presented.

ATTILA HÁZY: *On approximate Hermite–Hadamard type inequality* (Joint work with Judit Makó)

The main results of this paper offer sufficient conditions in order that an approximate lower Hermite–Hadamard type inequality implies an approximate Jensen convexity property. The key for the proof of the main result is a Korovkin type theorem.

In this paper we examine the implication from an upper Hermite–Hadamard type inequality to a Jensen type inequality. Thus in this paper, we are searching connections between the approximate upper Hermite–Hadamard inequality

$$(1) \qquad \int_{[0,1]} f(tx + (1-t)y)d\mu(t) \leq \lambda f(x) + (1-\lambda)f(y) + \alpha_H(x-y)$$

and the approximate Jensen inequality

$$(2) \qquad f\Big(\frac{x+y}{2}\Big) \leq \frac{f(x)+f(y)}{2} + \alpha_J(x-y) \qquad (x,y \in D),$$

where $f\colon D \to \mathbb{R}$, $\alpha_H, \alpha_J\colon D^* \to \mathbb{R}$ are given even functions, $\lambda \in \mathbb{R}$ and μ is a Borel probability measure on $[0,1]$.

ANTAL JÁRAI: *Baire property implies continuity for product and product of differences type functional equation*

It is proved that for product of unknown functions equal product of unknown functions type functional equations satisfied except a set of first category Baire property implies continuity and hence differentiability infinitely many times. A similar result is proved for the functional equation

$$\Big(f\big(t(x+y)\big) - f(tx)\Big)\Big(f(x+y) - f(y)\Big)$$
$$= \Big(f\big(t(x+y)\big) - f(ty)\Big)\Big(f(x+y) - f(x)\Big).$$

RAFAŁ KUCHARSKI: *On optimal dividends*

We consider the following classical problem of finance: what is the optimal dividend strategy that maximizes the expectation of the discounted

dividends to the shareholders (until possible ruin) in a stock company that is involved in risky business? Problem goes back at least to a 1957 paper of Bruno de Finetti presented to the International Congress of Actuaries, where a simple version of Cramér-Lundberg model of insurance company was considered. Since then a vast number of papers have dealt with that problem in many different variants, however there are still some unanswered questions to the general problem. During the talk we will show mathematical formulation of two versions of the problem of optimal dividends in discrete time setting, introduce underlying Bellman equation and present some known results, techniques and open problems.

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RADOSŁAW ŁUKASIK: *Some generalization of Cauchy's and Wilson's functional equations on abelian groups*

In the present talk, we consider the functional equation

$$\sum_{\lambda \in K} f(x + \lambda y) = |K|g(x) + \alpha(x)h(y), \quad (x, y \in G),$$

where $(G, +)$ is an abelian group, K is a finite, abelian subgroup of the automorphism group of G, X is a linear space over the field $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, $f, g, h \colon G \to X$, $\alpha \colon G \to \mathbb{K}$. We give the form of the solutions of the above functional equation (under some assumptions).

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JUDIT MAKÓ: *On approximate convexity* (Joint work with Attila Házy)

Let D be a nonempty convex subset of the normed space X and denote by D^* the set $\{\|x - y\| : x, y \in D\}$. Let $c > 0$ and $\alpha\colon D^* \to \mathbb{R}$ be a given continuous error function such that $\alpha(0) = 0$. We say that $f\colon D \to \mathbb{R}$ is (c, α)-Jensen convex, if, for all $x, y \in D$,

$$f\left(\frac{x+y}{2}\right) \leq cf(x) + cf(y) + \alpha(\|x - y\|).$$

In this report, we are looking for functions $\varphi\colon]0, 1[\to \mathbb{R}$ and $T_\alpha^c\colon]0, 1[\times D^* \to \mathbb{R}$, such that, for all $t \in]0, 1[$ and $x, y \in D$, the locally upper bounded, (c, α)-Jensen convex function f satisfies the following convexity type inequality:

$$f(tx + (1 - t)y) \leq \varphi(t)f(x) + \varphi(1 - t)f(y) + T_\alpha^c(t, \|x - y\|).$$

GYULA MAKSA: *On multiplicative Cauchy differences that are Hosszú differences* (Joint work with Roman Ger and Maciej Sablik)

In this talk, we present some results on the functional equation

(1) $g(x)g(y) - g(xy) = h(x + y - xy) - h(x) - h(y) + h(xy)$

where $g, h\colon [0, 1] \to \mathbb{R}$ and (1) holds for all $x, y \in [0, 1]$. It has turned out that (1) is not equivalent with the system

$$g(x)g(y) = g(xy) \qquad (x, y \in [0, 1])$$

$$h(x + y - xy) + h(xy) = h(x) + h(y) \qquad (x, y \in [0, 1]),$$

that is, the 'alienation' does not hold in the case of equation (1). Thus we focused on solving (1) when the function Γ is defined on $[0, 1]^2$ by $\Gamma(x, y) = g(x)g(y) - g(xy)$ is not identically zero.

We report on the results we got. A conjecture and open problems will also be presented.

LAJOS MOLNÁR: *On the standard K-loop structure of positive invertible elements in a C^*-algebra* (Joint work with Roberto Beneduci)

We show that the commutativity, the associativity and the distributivity of the operation $a \circ b = \sqrt{a}b\sqrt{a}$ on the set of all positive invertible elements of a C^*-algebra $\mathcal{A}$ are all equivalent to the commutativity of $\mathcal{A}$. We also present abstract characterizations of the operation $\circ$ and a few related ones too.

JANUSZ MORAWIEC: *On a functional equation involving iterates and powers*

Motivated by [1–8] we discuss the problem of the existence of continuous solutions $f\colon (0, +\infty) \to (0, +\infty)$ of the equation

$$f^2(x) = \gamma[f(x)]^\alpha x^\beta,$$

where α, β and γ are given reals. We show how to use [9] to solve the problem.

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GERGŐ NAGY: *Maps preserving the p-norm of linear combinations of positive operators*

In this talk, we describe the structure of those transformations on certain sets of positive operators which preserve the p-norm of linear combinations with given nonzero real coefficients. These sets are the collection of all positive pth Schatten-class operators and the set of its normalized elements. The results of the presentation generalize, extend and unify several former theorems.

ANDRZEJ OLBRYŚ: *On delta Schur-convex mappings*

In our talk, following Veselý and L. Zajiček [2], we introduce and investigate delta Schur-convex maps as a natural generalization of delta Schur-convex functions. Our main result establish a characterization of mappings generating delta Schur-convex sums. This result generalizes a well-known theorem of Ng [1] concerning Schur-convex sums.

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ZSOLT PÁLES: *Convexity with respect to families of means* (Joint work with Gyula Maksa)

Given a two-variable mean $M \colon \mathbb{R}_+^2 \to \mathbb{R}$, a function $f \colon \mathbb{R}_+ \to \mathbb{R}_+$ is called M-convex if, for all $x, y \in \mathbb{R}_+$,

$$f(M(x,y)) \leq M(f(x), f(y)).$$

Denote by H_p the p-th Hölder (or power) mean. For $p > 0$, the H_p-convexity of f is equivalent to the Jensen-convexity of the function f_p defined by $f_p(t) := \left(f\left(t^{\frac{1}{p}} \right) \right)^p$. Thus, in general, the H_p-convexity of a function does not imply its continuity.

We investigate the simultaneous validity of "many" H_p-convexity properties and we show the following two statements:
— There exists a discontinuous function which is H_p-convex for all rational $p > 0$.
— If a function is H_p-convex for all $p \in P$, where $P \subseteq \mathbb{R}_+$ is a set with positive inner Lebesgue measure then it is continuous.

BARBARA PRZEBIERACZ: *Dynamical systems and their stability*

The classic definition of dynamical system (DEFINITION 1) reads as follows: the continuous function $F \colon \mathbb{R} \times I \to I$, is called *a dynamical system* if *the translation equation*:

(1) $$F(t, F(s, x)) = F(t + s, x) \ \text{ for } t, s \in \mathbb{R}, \ x \in I$$

as well as *the identity condition*:

(2) $F(0, x) = x$ for $x \in I$

are satisfied. But there are also other systems of equations, which are equivalent to system (1) & (2). For example, dynamical systems may also be defined equivalently in the following way (DEFINITION 2): the continuous function $F \colon \mathbb{R} \times I \to I$ is called a dynamical system if F is a solution of the translation equation such that

(3) $F'(0, x) = 1$ for $x \in I$.

Let $\mathcal{K}_1$ be the class of all continuous functions $F \colon \mathbb{R} \times I \to I$ such that $F(0, \cdot)$ is strictly increasing, let $\mathcal{K}_2$ be the class of all continuous functions $F \colon \mathbb{R} \times I \to I$ such that $F'(0, \cdot)$ exists, let $\mathcal{K}_3$ be the class of all continuous functions $F \colon \mathbb{R} \times I \to I$ such that F is surjection.

Other equivalent definitions of the dynamical systems are:

DEFINITION 3. The solution of the translation equation $F \colon \mathbb{R} \times I \to I$, such that $F \in \mathcal{K}_1$, is called a dynamical system.

DEFINITION 4. The not-constant function $F \colon \mathbb{R} \times I \to I$ is called the dynamical system if F is a solution of translation equation (1) and $F \in \mathcal{K}_2$.

DEFINITION 5. The $F \colon \mathbb{R} \times I \to I$, $F \in \mathcal{K}_3$, which satisfies the translation equation (1), is called a dynamical system.

We consider the b-stability, uniform b-stability, the inverse stability, the inverse b-stability, the inverse uniform b-stability, the superstability, the inverse superstability for the five given definitions of dynamical system. The results are summarized in the table.

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	def. 1 ((1) & $F(0,x)=x$)	def. 2 ((1) & $F'(0,x)=1$)	def. 3 ((1) & $F(0,\cdot)$ strictly increasing)	def. 4 ((1) & $F'(0,x)$ exists)	def. 5 ((1) & F surjection)
Ulam-Hyers stability	only for $I=\mathbb{R}$	for every I	for no I		for every I
b-stability uniform b-stability	only for I bounded or $I=\mathbb{R}$	for every I	only for I bounded		for every I
inverse stability	for no I				
inverse b-stability inverse uniform b-stability	only for I bounded	for no I	only for I bounded		
superstability	only for I bounded				
inverse superstability	only for I bounded	for no I	for every I		
hiperstability	for no I				
inverse hiperstability	for every I				

Maciej Sablik: *A misleading multiplied number of independent variables* (Joint work with Roman Ger)

In a booklet by Y.S. Brodsky and A.K. Slipenko [1] the following two problems were formulated.

A. (Problem 5, p. 22) Find all continuous functions defined in $(0,\infty)$ and satisfying the equation

$$(1) \qquad f(f(x)) = xf(x).$$

B. (Exercise 4 a), p. 66) Determine solutions of the functional equation

$$(2) \qquad f(f(x)) = f(x) + x,$$

in the class of differentiable functions mapping $\mathbb{R}$ into $\mathbb{R}$.

We solve both problems and show that

A. has been wrongly solved in [1];

B. does not require differentiability assumption, continuity is enough.

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JUSTYNA SIKORSKA: *Set-valued orthogonally additive functions*

A single-valued orthogonally additive function from an orthogonality space into an Abelian group is of the form $a+q$, where a is additive and q is quadratic (see J. Rätz [3], and also [1], [2]).

We give solutions of the equation of orthogonal additivity in the class of set-valued functions defined on an orthogonality space.

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PATRÍCIA SZOKOL: *Transformations on positive definite matrices preserving generalized distance measures* (Joint work with Lajos Molnár)

The aim of this presentation is to extend and unify former results on the structure of surjective isometries of spaces of positive definite matrices obtained in a paper of Lajos Molnár. The isometries there correspond to certain geodesic distances in Finsler-type structures and to a recently defined interesting metric which also follows a non-Euclidean geometry. In our new results we consider not only true metrics but so-called generalized distance measures which are parameterized by unitarily invariant norms and continuous real functions satisfying certain conditions. We also present results concerning similar preserver transformations defined on the subset $\mathbb{P}_n^c$ of all positive definite matrices with constant determinant c. In fact, following the approach given in the mentioned paper we shall determine the structure of all continuous Jordan triple endomorphisms of $\mathbb{P}_n^1$ (i.e. continuous maps respecting the Jordan triple product ABA) and then we describe the surjective maps of $\mathbb{P}_n^1$ that leave a given distance measure invariant.

TOMASZ SZOSTOK: *Ohlin's lemma and some inequalities of the Hermite–Hadamard type*

Inspired by a recent paper of T. Rajba [2], we use the Ohlin lemma (see [1]) to obtain some inequalities of the Hermite–Hadamard type. Namely, we find all numbers $a, \alpha, \beta \in [0,1]$ such that for all convex functions $f \colon [x,y] \to \mathbb{R}$, the inequality

$$af(\alpha x + (1-\alpha)y) + (1-a)f(\beta x + (1-\beta y)) \leq \frac{1}{y-x} \int_x^y f(t)dt$$

is satisfied. Similarly, we determine all $b, c, d, \gamma \in (0, 1)$ with $b + c + d = 1$ such that the inequality

$$bf(x) + cf(\gamma x + (1 - \gamma)y) + df(y) \geq \frac{1}{y - x} \int_x^y f(t)dt$$

is satisfied for every convex f.

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PAWEŁ WÓJCIK: *On a some root and an invariant subspace*

Let M be Banach space. The Banach space of all bounded linear operators from M to M is denoted by $B(M)$. The aim of this report is to discuss an invariant subspace of some surjective operator.

If A is a surjective bounded operator, then A could not possess a square root, i.e., the functional equation $X^2 = A$ could not be solved. But, there exists nontrivial subspace $L \subset M$ such that $A|_L \colon L \to L$ has a square root, i.e., there is $T \in B(L)$ such that $T^2 = A|_L$. In particular, the subspace L is invariant for A. Moreover, the invariant subspace L may be found so that $A|_L$ is invertible. A similar result is true also for the functional equation $X^n = A$.

2. Problems and Remarks

1. PROBLEM   Real additive functions may satisfy further functional equations, for instance, there exist nontrivial (i.e., nonzero) additive functions that satisfy the so-called Leibniz Rule; these functions are termed derivations. This note is about multiplicative functions which are Jensen-convex simultaneously. The following result characterizes such functions in terms of a functional inequality.

PROPOSITION. Let $m \colon \mathbb{R}_+ \to \mathbb{R}$ be a nonzero multiplicative function, i.e., let m satisfy $m(xy) = m(x)m(y)$ for $x, y > 0$. Then m is Jensen convex if and only if there exists an additive function $a \colon \mathbb{R} \to \mathbb{R}$ such that

$$(1) \qquad\qquad m(t) \geq 1 + a(t - 1) \qquad (t > 0).$$

PROOF. If m is a nonzero multiplicative function, then m is positive everywhere and $m(1) = 1$.

If m is Jensen convex then, as a consequence of Rodé's Theorem [3], for every $p > 0$ there exists an additive function $a_p \colon \mathbb{R} \to \mathbb{R}$ such that

$$m(t) \geq m(p) + a_p(t - p) \qquad (t > 0).$$

Therefore, (1) holds with $a = a_1$.

No assume that there exists an additive function $a \colon \mathbb{R} \to \mathbb{R}$ satisfying (1). Let $x, y > 0$ and apply (1) for $t := \frac{2x}{x+y}$ and $t := \frac{2y}{x+y}$. Adding up the inequalities so obtained side by side, we get

$$m\Big(\frac{2x}{x+y}\Big) + m\Big(\frac{2y}{x+y}\Big) \geq 2 + a\Big(\frac{2x}{x+y} - 1\Big) + a\Big(\frac{2y}{x+y} - 1\Big)$$
$$= 2 + a\Big(\frac{2x}{x+y} + \frac{2y}{x+y} - 2\Big) = 2.$$

Now multiplying this inequality by $m\big(\frac{x+y}{2}\big)$ and using the multiplicativity of m, it follows that

$$m(x) + m(y) \geq 2m\Big(\frac{x+y}{2}\Big),$$

which proves that m is Jensen convex. $\qquad\qquad\qquad\qquad\qquad\qquad\square$

The continuous nonzero multiplicative functions are of the form $m(x) = x^p$, where p is a real constant. It is easy to check (for instance, using the standard second-derivative test of convexity) that a power function $m(x) = x^p$ is Jensen convex (if and only if it is convex) if and only if $p \in\,]-\infty, 0] \cup [1, \infty[$. Thus, continuous Jensen convex multiplicative functions can completely be described. On the other hand, there exist discontinuous Jensen convex and multiplicative functions. The existence of such a function was asked by Janusz Matkowski [2] and this question was answered in the positive by Gyula Maksa [2] during the 30th International Symposium on Functional Equations in 1992 in Oberwolfach. Gyula Maksa proved that, given any derivation $d \colon \mathbb{R} \to \mathbb{R}$, the function $m \colon \mathbb{R}_+ \to \mathbb{R}$ defined by

$$m(x) := x \exp\Big(\frac{d(x)}{x}\Big) \qquad (x > 0)$$

is multiplicative and Jensen convex. In my talk, a more general statement was shown: For any $p \geq 1$, and any subadditive function $d \colon \mathbb{R} \to \mathbb{R}$ which satisfies the Leibniz Rule, the function $m \colon \mathbb{R}_+ \to \mathbb{R}$ defined by

$$m(x) := x^p \exp\Big(\frac{d(x)}{x}\Big) \qquad (x > 0)$$

is multiplicative and Jensen convex. More generally, it turns out that these functions are convex with respect to all Hölder means with a positive rational parameter.

The *open problem* is to characterize those pairs (a, m) of additive and multiplicative functions that satisfy (1). Do a and m mutually determine each other?

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<div align="right">ZSOLT PÁLES</div>

2. REMARK *(Remark to the regularity of functions satisfying a multiplicative functional equation almost everywhere)* An example where we can apply our method in [3] and the methods in my talk (because the key observations remain true) to prove that if the functional equation is satisfied almost everywhere and on one side none of the unknown real-valued functions is almost everywhere zero then all are almost everywhere nonzero and they are almost equal to $\mathcal{C}^{\infty}$-functions satisfying the equation everywhere is the functional equation

$$f_1(x_1)f_2(x_2)f_3(x_3) = g_1(y_1)g_2(y_2)g_3(y_3) \quad (0 < x_1, x_2, x_3 < 1),$$

where

$$y_1 = x_2 + (1 - x_1)(1 - x_2)x_3,$$

$$y_2 = \frac{x_1 x_2}{x_2 + (1 - x_1)(1 - x_2 x_3)}$$

and

$$y_3 = \frac{x_1 x_3}{\big(1 - (1 - x_1)x_3\big)\big(x_2 + x_3 - x_2 x_3\big)}.$$

The Baire category case can be treated, too. About this equation see [1] of Bobecka and Wesołowski.

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<div align="right">Antal Járai</div>

3. Problem   Hermite–Hadamard inequality may be written in the following way

$$(1) \qquad f\left(\frac{x+y}{2}\right) \le \frac{F(y) - F(x)}{y - x} \le \frac{f(x) + f(y)}{2}.$$

It is easy to obtain solutions of this inequality if we assume that f satisfies any regularity condition which forces a Jensen convex function to be convex. However equation (1) as it stands may be considered without any regularity assumptions. Therefore it is natural to ask for a general solution of (1).

<div align="right">Tomasz Szostok</div>

4. Remark *(On mathability)*   The topic of this note is related to the concept of mathability introduced in the paper [6]. Our aim is to point out some connections between mathability and functional equations and inequalities.

Mathability is considered to be a branch of cognitive infocommunications (CogInfoCom) that investigates any combination of artificial and natural cognitive capabilities relevant to mathematics, including a wide spectrum of areas ranging from low-level arithmetic operations to high-level symbolic reasoning. Investigations on mathability extend to the question of how artificial mathematical capabilities can be quantified. Further, an important goal of mathability is to develop a set of methodologies using which human mathematical capabilities can be emulated and enhanced. (Concerning some further investigations of mathability, we refer to the paper [12], too, basic facts on cognitive infocommunications can be found, among others, in [4] and [5].)

In the following we consider questions related to the mathability of some systems. Mathematical capabilities of existing systems can be used for solving mathematical problems in the following forms:

– applying existing functions and methods provided by a system to solve problems,

— some tools provided by the system are contained in the final solution of the problem (e.g., they are contained in mathematical proofs),

— the mathematical capabilities of the system are used to get ideas only,

– developing new programs or functions for solving open problems.

We discuss these possibilities and present some examples related to them. Our examples are connected to investigation of functional equations and inequalities. The reason for choosing this field is, that the solution of functional equations and inequalities with computers requires the existence of high-level symbolic operations of the underlying system, or, using our terminology we may say that it is connected to a 'high-level of mathability'. The investigations we mention in our examples are all related to the computer algebra system Maple (which is a registered trade mark of Waterloo Maple Inc.). A more detailed description of our examples is given in [6].

At first, we consider the case when existing tools of a given system are used for answering open questions.

In several situations, when solving practical or even theoretical problems, mathematical tools of a system are used to make complicated and tiresome calculations via computer (similarly to the usage of a simple calculator for arithmetic operations). In their papers [1], [2], [3], Sz. Baják and Zs. Páles investigated a so called invariance equation for various homogeneous means. Using transformations, differentiations and other methods, they obtained very complicated systems of polynomial equations for parameters. They handled some part of their computations via computers (performing symbolic manipulations in Maple) and they got some simple connections between the parameters. It is easy to see that in their calculations numerical methods (more precisely, computer programs, using numerical methods) could not be applied. It is also important to mention that with the help of such computations, exact pure mathematical results were obtained and proved.

Another possibility in this category is when computers are used, 'to get ideas' in connection with unsolved theoretical problems. Nowadays, this method is used more and more often and this usage leads to a so called 'experimental mathematics'. In this case, computations done by a computer, or 'the mathability of the system' which was used for getting some ideas do not appear in the final form of the solutions at all. However, it is obvious that the 'level of the mathability' of the system applied plays a crucial role in such 'experiments'.

Finally, we consider the situation when new computer programs are developed for solving problems. In this case, the programs can be the 'main products' of the solution process and, in most of the cases, they also 'improve the mathability' or 'increase the level of the mathability' of the underlying system. As examples, we may consider some programs developed in Maple for solving linear functional equations and systems of linear functional equations presented in the papers [7], [8], [9], [10], and [11], respectively.

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