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Report of Meeting

The Eighth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities Fűzfa Pihenőpark, Poroszló (Hungary) January 30 – February 2, 2008

The Eighth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities was held in Poroszló, Hungary, from January 30 to February 2, 2008, at Fűzfa Pihenőpark. It was organized by the Institute of Mathematics of the University of Debrecen, with the financial support of the Hungarian Scientific Research Fund OTKA T-043080.

24 participants came from the University of Debrecen (Hungary) and the Silesian University of Katowice (Poland) at 12 from each of both cities.

Professor Zsolt Páles opened the Seminar and welcomed the participants to Poroszló.

The scientific talks presented at the Seminar focused on the following topics: equations in a single variable and in several variables, iteration theory, equations on algebraic structures, regularity properties of the solutions of certain functional equations, functional inequalities, Hyers–Ulam stability, functional equations and inequalities involving mean values, generalized convexity. Interesting discussions were generated by the talks.

There were three very profitable Problem Sessions.

The social program included a bowling competition and a banquet. The closing address was given by Professor Roman Ger. His invitation to the Ninth Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities in February 2009 in Poland was gratefully accepted.

Summaries of the talks in alphabetic order of the authors follow in Section 1, problems and remarks in chronological order in Section 2, and the list of participants in the final section.

1. Abstract of talks

Roman Badora: *Invariant means and intersection properties*

Let S be a semigroup and let $\mathcal{C}$ be a family of subsets of a set Y. We say that a function $f : S \to Y$ belongs to the space $B^{\mathcal{C}}(S, Y)$ iff the set $\mathcal{C}_f$ of all $C \in \mathcal{C}$ such that $f(S) \subset C$ is non-empty.

In the talk we study connections between binary and finite intersection properties of the family $\mathcal{C}$ and the existence of invariant means on the space $B^{\mathcal{C}}(S, Y)$.

Szabolcs Baják: *Further results in solving functional equations with computer* (Joint work with A. Gilányi)

In 1982 László Székelyhidi gave the solution of general linear functional equations in his paper *On a class of linear functional equations.* In 1995 in his PhD thesis, Attila Gilányi created a computer program which implemented these theoretical results. The aim of our talk is to present the first steps of the development of this program which, as is our purpose, may in the future become a Maple package itself.

Mihály Bessenyei: *The Markov–Krein representation problem of Beckenbach families*

Beckenbach families are sets of continuous functions fulfilling a unique interpolation property [3]. The most important examples for Beckenbach families are the *Chebyshev systems*, that posses an additional linear structure.

According to the theory of Markov and Krein [2], each Chebyshev system has both lower and upper *principal representations*; that is, the integral of each function belonging to the system can be represented as the linear combination of the function's values taken at certain base points. The number of base points is approximately the half of the dimension of the underlying Chebyshev system, and themselves the points depend only on the system, too.

The aim of the talk is to investigate the existence of principal representations in the case of Beckenbach families. It turns out (see [1]) that there exist Beckenbach families that have neither lower, nor upper principal representation; contrary, there exist Beckenbach families that have lower (respectively, upper) principal representation.

References

[1] Bessenyei M., *The Hermite–Hadamard inequality in Beckenbach's setting*, manuscript.

[2] Karlin S., Studden W.J., *Tchebycheff systems: With applications in analysis and statistics*, Pure and Applied Mathematics, Vol. XV, Interscience Publishers John Wiley & Sons, New York-London-Sydney, 1966.
[3] Popoviciu T., *Les fonctions convexes*, Hermann et Cie, Paris, 1944.

Zoltán Boros: *A remark on the estimations of approximate convexity*

Let $p \in]1, \infty[$ and

$$\Phi_p(x) = 2^p \sum_{n=0}^{\infty} \frac{(\mathrm{dist}(2^n x, \mathbb{Z}))^p}{2^n}, \quad x \in \mathbb{R}.$$

As it is stated in the following result of Házy and Páles (cf. [1, Theorem 6]), the function Φ_p plays a specific role in the theory of approximately convex functions.

Theorem 1. *Let $I \subset \mathbb{R}$ be an open interval and $\varepsilon \geq 0$. If $f : I \to \mathbb{R}$ fulfils the inequality*

$$f\left(\frac{x+y}{2}\right) \leq \frac{f(x)+f(y)}{2} + \varepsilon|x-y|^p \tag{1}$$

for every $x, y \in I$ and f is locally bounded from above at a point, then f is locally bounded and the inequality

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y) + \varepsilon \Phi_p(\lambda)|x-y|^p \tag{2}$$

holds for all $x, y \in I$ and $\lambda \in [0, 1]$.

In order to replace the coefficient $\Phi_p(\lambda)$ in (2) with a simpler expression, one may wish to determine the asymptotic magnitude of the function Φ_p around zero. This problem is left open by the authors [1, Remark 1]. Concerning this problem, we establish the following remark.

Theorem 2. *Either*
(A) *for every $q > 1$, $K > 0$, and $\delta > 0$ there exists a $\lambda \in]0, \delta[$ such that*

$$\Phi_p(\lambda) > K\lambda^q,$$

or the following implication holds:
(B) *if $I \subset \mathbb{R}$ is an open interval, $\varepsilon \geq 0$, and $f : I \to \mathbb{R}$ satisfies the assumptions of Theorem 1, then f is convex.*

Reference

[1] Házy A., Páles Zs., *On approximately t-convex functions*, Publ. Math. Debrecen **66** (2005), no. 3–4, 489–501.

Pál Burai: *On a class of homogeneous means*

We examine the equality, translativity and comparability problem in the class of Daróczy means:

$$\mathcal{D}_{\alpha,p}(x,y) := \begin{cases} \left(\frac{x^p+\alpha(\sqrt{xy})^p+y^p}{\alpha+2}\right)^{1/p}, & \text{if } p \neq 0\,,\ -1 \le \alpha < \infty, \\ \mathcal{G}(x,y) := \sqrt{xy}, & \text{if } p = 0 \text{ or } \alpha = \infty, \end{cases} \qquad x,y \in \mathbb{R}_+\,.$$

Zoltán Daróczy: *On a family of functional equations with one parameter*

Let $I \subset \mathbb{R}$ be a non-void open interval and let $0 < \alpha < 1$, $\alpha \neq \frac{1}{2}$ be a given parameter. The functions $f, g : I \to \mathbb{R}_+$ are solutions of the functional equation

$$f\left(\frac{x+y}{2}\right)\Big(2\alpha g(y) - g(x)\Big) = \alpha f(x)g(y) - (1-\alpha)f(y)g(x), \quad x, y \in I,$$

if and only if f and g are constant functions on I.

Włodzimierz Fechner: *On a separation for the Cauchy equation on spheres* (Joint work with J. Sikorska)

We give sufficient conditions for separating functions p and q, satisfying for a given function φ the conditional inequalities

$$\varphi(x) = \varphi(y) \Longrightarrow p(x+y) \le p(x) + p(y),$$
$$\varphi(x) = \varphi(y) \Longrightarrow q(x+y) \ge q(x) + q(y),$$

by an additive mapping.

Roman Ger: *Additivity and exponentiality are alien each to the other*

Let $(X, +)$ be a unitary ring and let $(Y, +, \cdot)$ an integral domain. We study the solutions of a Pexider type functional equation

$$f(x+y) + g(x+y) = f(x) + f(y) + g(x)g(y)$$

for functions f and g mapping X into Y. Our chief concern is to examine whether or not this functional equation is equivalent to the system of two Cauchy equations

$$\begin{cases} f(x+y) = f(x) + f(y) \\ g(x+y) = g(x)g(y) \end{cases}$$

for every $x, y \in X$.

Attila Gilányi: *Stability of a functional equation characterizing the absolute value of additive functions* (Joint work with Kaori Nagatou and Peter Volkmann)

In the present talk the stability of the functional equation

$$\max\{f(x \circ y^2), f(x)\} = f(x \circ y) + f(y), \quad x, y \in S,$$

is proved for real valued functions defined on a square-symmetric groupoid $(S, \circ)$ with a left unit element.

In the case when $(S, \circ) = (G, +)$ is an abelian group, this equation is equivalent to that of

$$\max\{f(x+y), f(x-y)\} = f(x) + f(y), \quad x, y \in G.$$

It was proved in the paper [1] by Alice Simon and Peter Volkmann that the solutions of this equation are of the form $f(x) = |a(x)|$, $(x \in G)$, where a is a real valued additive function defined on G. The stability of this equation was shown during the 45^{th} International Symposium on Functional Equations, Bielsko–Biała, Poland, in 2007 by Peter Volkmann (cf. [2]).

References

[1] Simon A., Volkmann P., *Caractérisation du module d'une fonction additive à l'aide d'une équation fonctionelle*, Aequationes Math. **47** (1994), 60–68.
[2] Volkmann P., *Stability of functional equations for the absolute value of additive functions*, (Abstract, Report of Meeting) Aequationes Math. 75 (2008), 180.

Eszter Gselmann: *Stability type results concerning the fundamental equation of information of multiplicative type*

In this talk we shall deal with the stability of the fundamental equation of information of multiplicative type and we will prove the following.

Theorem. *Let $\varepsilon \geq 0$ be arbitrary, $M : [0,1]^k \to \mathbb{R}$ be multiplicative but not additive and $f : [0,1]^k \to \mathbb{R}$ be a function. Assume that*

$$\left| f(x) + M(\mathbf{1} - x) f\left(\frac{y}{\mathbf{1} - x}\right) - f(y) - M(\mathbf{1} - y) f\left(\frac{x}{\mathbf{1} - y}\right) \right| \leq \varepsilon \tag{1}$$

holds for all $(x,y) \in \left\{(x,y) \in \mathbb{R}^{2k} | x, y \in [0,1[^k, x + y \leq \mathbf{1}\right\}$. Then there exist constant $a, b \in \mathbb{R}$ and $K \in \mathbb{R}$ depending only on the function M such that

$$|f(x) - (aM(x) + b(M(\mathbf{1} - x) - 1))| \leq K\varepsilon \tag{2}$$

holds for all $x \in [0,1]^k$.

Gyula Maksa: *Hyperstability of a functional equation*

In this talk we present the following

Theorem. *Let $\alpha, \varepsilon \in \mathbb{R}$ be fixed, $\alpha < 0$ and $\varepsilon \geq 0$. Then the function $f :]0,1[\to \mathbb{R}$ satisfies the inequality*

$$\left| f(x) + (1-x)^\alpha f\left(\frac{y}{1-x}\right) - f(y) - (1-y)^\alpha f\left(\frac{x}{1-y}\right) \right| \leq \varepsilon$$

for all $(x,y) \in D \doteq \{(x,y) \in \mathbb{R}^2 : x, y, x+y \in]0,1[\}$ if and only if there exist $a, b \in \mathbb{R}$ such that

$$f(x) = ax^\alpha + b((1-x)^\alpha - 1), \quad x \in]0,1[. \tag{1}$$

For any $a, b \in \mathbb{R}$, the function f defined in (1) is a solution of the equation

$$f(x) + (1-x)^\alpha f\left(\frac{y}{1-x}\right) = f(y) + (1-y)^\alpha f\left(\frac{x}{1-y}\right), \quad (x,y) \in D.$$

This is the reason why we say that this equation is hyperstable.

Janusz Matkowski: *A converse of Hölder's inequality theorem*

Let (Ω, Σ, μ) be a measure space such that $0 < \mu(A) < 1 < \mu(B) < \infty$ for some $A, B \in \Sigma$ and let bijections $\varphi_1, \varphi_2, \psi_1, \psi_2 : (0,\infty) \to (0,\infty)$ be such that

$$\frac{\psi_1 \circ \varphi_1(t)}{t} \leq c \leq \frac{t}{\psi_2 \circ \varphi_2(t)}, \quad t > 0.$$

We prove that if

$$\int_{\Omega} xyd\mu \leq \psi_1\left(\int_{\Omega(\mathbf{x})} \varphi_1 \circ |x|\, d\mu\right)\psi_2\left(\int_{\Omega(\mathbf{y})} \varphi_2 \circ |y|\, d\mu\right)$$

for all nonnegative μ-integrable simple functions $\mathbf{x}, \mathbf{y} : \Omega \to \mathbb{R}$ (where $\Omega(\mathbf{x})$ stands for the support of $\mathbf{x}$), then there exists a real $p > 1$ such that

$$\frac{\varphi_1(t)}{\varphi_1(1)} = t^p, \quad \frac{\psi_1(t)}{\psi_1(1)} = t^{1/p}, \quad \frac{\varphi_2(t)}{\varphi_2(1)} = t^q, \quad \frac{\psi_2(t)}{\psi_2(1)} = t^{1/q}, \quad t > 0,$$

where $\frac{1}{p} + \frac{1}{q} = 1$. A relevant result for the reversed inequality is also given.

Fruzsina Mészáros: *A characterization of the exponential distribution through functional equations* (Joint work with Gy. Maksa)

We discuss the following problem. Find all density functions f satisfying the following two properties

Property 1. $f(u) = 0$ *for almost all* $u \in\,]-\infty, 0\,[$ (*with respect to the Lebesgue measure*) and

Property 2. *There exist* $0 \leq n \in \mathbb{Z}$ (*the set of all integers*) *and* $-1 < \beta \in \mathbb{R}$ (*the set of all real numbers*) *such that the function* p *defined on* $\mathbb{R}^2$ *by* $p(u, v) = 0$ *if* $u < 0$ *or* $v < 0$ *and*

$$p(u,v) = \int_0^{+\infty} f(u)\left(F(u) - F(s+u)\right)^n f(s+u) f(s+u+v) F(s+u+v)^{\beta} ds$$

if $u, v \in [\,0, +\infty\,[$, *where* $F(u) = \int_u^{+\infty} f$, $u \geq 0$, *is the survival function, is the joint density function of some two independent random variables.*

In this talk we give a solution of the problem by proving that all density functions f which are positive on $[\,0, +\infty\,[$ and have Properties 1–2 are exponential density functions.

Lajos Molnár: *Maps on positive operators preserving operator means*

Let H be a complex Hilbert space with inner product $\langle .,. \rangle$ and denote $B(H)$ the algebra of all bounded linear operators on H. An operator $A \in B(H)$ is said to be positive if $\langle Ax, x\rangle \geq 0$ for all $x \in H$. The symbol $B(H)^+$ stands for the set of all positive operators on H. In 1978, Ando defined the geometric and harmonic means for pairs of positive operators. We denote these operations by # and !, respectively. Moreover, we also consider the parallel sum : and the arithmetic mean $\triangledown$ on $B(H)^+$.

In this talk for any $\sigma \in \{\#, !, :, \triangledown\}$ we present the structure of all bijective maps $\phi : B(H)^+ \to B(H)^+$ which have the property

$$\phi(A\sigma B) = \phi(A)\sigma\phi(B), \quad A, B \in B(H)^+.$$

Janusz Morawiec: *On L^1-solutions of a two-direction refinement equation*

Fix integers $k \geq 2$, $N \geq 1$ and a matrix $\begin{bmatrix} c_{-N,-1} & c_{-N+1,-1} & \cdots & c_{N,-1} \\ c_{-N,1} & c_{-N+1,1} & \cdots & c_{N,1} \end{bmatrix}$ of nonnegative reals such that

$$\sum_{n\in\{-N,\dots,N\}} \sum_{\varepsilon\in\{-1,1\}} c_{n,\varepsilon} = k.$$

We prove that the vector space of all L^1-solutions of the following two-direction refinement equation

$$f(x) = \sum_{n=-N}^{N} c_{n,1} f(kx-n) + \sum_{n=-N}^{N} c_{n,-1} f(-kx+n) \tag{1}$$

is at most one dimensional and consists of compactly supported functions of constant sign. We also show that in many interesting cases any L^1-solution of (1) is either positive or negative on its support. Next we present sufficient conditions (easy for verification) for the existence of nontrivial L^1-solutions of (1) as well as for the nonexistence of such solutions.

Agata Nowak: *Pexider equation on a restricted domain*

Let X be a linear topological space, G an abelian group and $D \subseteq X$ a nonempty set. Let $f\colon D_+ \to G$, $g\colon D_1 \to G$, $h\colon D_2 \to G$ where $D_+ := \{x + y : \quad (x, y) \in D\}$, $D_1 := \{x \in X : \exists_{y\in X} \quad (x, y) \in D\}$, $D_2 := \{y \in X : \exists_{x\in X} \quad (x, y) \in D\}$ fulfil the condition $f(x + y) = g(x) + h(y)$ whenever $(x, y) \in D$. The article "Pexider‘s equation and aggregation of allocations" of F. Rado and John A. Baker contains some results which enable extending the functions f, g, h to the function F, G, H respectively in such a way that $F(x + y) = G(x) + H(y)$ for all $x, y \in X$. Our talk will be devoted to some generalizations of these theorems.

Andrzej Olbryś: *The support theorem for t-Wright concave functions and its consequences*

Let $t \in (0, 1)$ be a fixed number, $L(t)$ be the smallest field containing the set $\{t\}$, and let X be a linear space over the field K, where $L(t) \subset K \subset \mathbb{R}$. Assume, moreover that $D \subset X$ is a $L(t)$-convex set, i.e., $\alpha D + (1-\alpha)D \subset D$, for all $\alpha \in L(t) \cap (0,1)$. A function $f\colon D \to \mathbb{R}$ is said to be a t-Wright convex iff

$$f(tx + (1-t)y) + f((1-t)x + ty) \leq f(x) + f(y), \quad x, y \in D, \tag{1}$$

it is said to be a t-Wright affine if the above inequality is satisfied with equality. If (1) is fulfilled for $t = \frac{1}{2}$ then f is called convex in the sense of Jensen. If $f\colon D \to \mathbb{R}$ is a function such that $-f$ is a t-Wright convex (Jensen-convex) then f is called t-Wright concave (Jensen-concave).

We present a theorem given a necessary and sufficient condition that for an arbitrary point $y \in X$ and a t-Wright concave function $f\colon X \to \mathbb{R}$ there exists a support function $a_y\colon X \to \mathbb{R}$ i.e. the function satisfying the following conditions:

(i) $a_y(tx + (1-t)z) + a_y((1-t)x + tz) = a_y(x) + a_y(z), \quad x, z \in X,$
(ii) $a_y(x) \leq f(x), \quad x \in X,$
(iii) $a_y(y) = f(y).$

As a consequences of this theorem we obtain some "strange" properties concerning t-Wright convex functions defined on the whole space X. In particulary, from our results it is follow that every t-Wright convex function which is bounded above on X is Jensen-concave. In the case where $t = \frac{1}{2}$ every such function must to be constant but there are such numbers $t \in (0,1)$ for which it is not true (see [1]). Obviously, the whole space X is not natural domain for t-Wright convex functions but in our consideration this assumption is essential and cannot be omitted.

Reference

[1] Maksa Gy., Nikodem K., Páles Zs., *Result on t-Wright convexity*, C.R. Math. Rep. Acad. Sci. Canada **13** (1991), 274–278.

Zsolt Páles: *Stability of homogeneity and multiplicativity of means*

The notions of homogeneity and multiplicativity are essential properties of means. By the standard definition, an n-variable mean $M\colon \mathbb{R}_+^n \to \mathbb{R}_+$ is said to be *homogeneous* if, for all $t, x_1, \dots, x_n \in \mathbb{R}_+$,

$$M(tx_1, \dots, tx_n) = tM(x_1, \dots, x_n).$$

A mean $M\colon \bigcup_{n=1}^{\infty} \mathbb{R}_+^n \to \mathbb{R}_+$ is called *homogeneous* if, for all $n \in \mathbb{N}$, its n-variable restriction $M|_{\mathbb{R}_+^n}$ is a homogenous mean on $\mathbb{R}_+^n$.

A mean $M\colon \bigcup_{n=1}^{\infty} \mathbb{R}_+^n \to \mathbb{R}_+$ is said to be *multiplicative* if, for all $n, m \in \mathbb{N}$, $x = (x_1, \dots, x_n) \in \mathbb{R}_+^n$, and $y = (y_1, \dots, y_m) \in \mathbb{R}_+^m$,

$$M(x * y) = M(x)M(y),$$

where $x * y := (x_1y_1, \dots, x_ny_1, \dots, x_1y_m, \dots, x_ny_m) \in \mathbb{R}_+^{nm}$.

It is obvious that multiplicativity implies homogeneity, however the reversed implication is not valid. One can easily see that the power (or Hölder) means and the Gini means are homogeneous, moreover multiplicative means.

The aim is to introduce the stability of these two key properties in an adequate manner and then to show that they are stable in the sense of (Pólya–Szegő–)Hyers–Ulam(–Aoki–Rassias–Gavruţa).

Maciej Sablik: *On means invariant with respect to binary operations*

This is a report on a joint work with Małgorzata Pałys. There is a large number of results concerning translative or homogeneous means (see eg. J. Aczél and J. Dhombres, *Functional Equations in Several Variables.* Cambridge University Press, Cambridge, 1989). In other words, those are results on means which are invariant with respect to addition or multiplication, respectively. We try to ask for invariance with respect to some other operations. Under suitable assumptions, we determine classes of binary, or n-ary means satisfying the assumption of invariance with respect to operations different from those aforementioned. We are also looking for bisymmetric means in function spaces which are translative, homogeneous, or invariant with respect to other operations.

Justyna Sikorska: *On a conditional Pexider functional equation*

We study a conditional Pexider functional equation and its stability on prescribed sets being generalizations of spheres, namely, we study an equation of the form

$$\gamma(x) = \gamma(y) \quad \text{implies} \quad f(x + y) = g(x) + h(y)$$

with given function γ and unknown functions f, g, h.

László Székelyhidi: *Spectral synthesis on Sturm–Liouville hypergroups*

This talk presents some recent problems concerning spectral synthesis on Sturm–Liouville hypergroups. Spectral analysis and spectral synthesis are considered on finite dimensional varieties.

Tomasz Szostok: *On the stability of the equation stemming from Lagrange MVT* (Joint work with Sz. Wąsowicz)

We deal with the stability of the equation

$$F(y)-F(x)=(y-x)f\left(\frac{x+y}{2}\right), \tag{1}$$

thus we consider the inequality

$$\left|F(y)-F(x)-(y-x)f\left(\frac{x+y}{2}\right)\right|<\varepsilon$$

and we show that if functions $f, F\colon \mathbb{R}\to\mathbb{R}$ satisfy this inequality, then f must be a solution of (1) (with $F(x)=xf\left(\frac{x}{2}\right)$).

Wirginia Wyrobek: *Orthogonally additive functions modulo a discrete subgroup*

Following K. Baron and P. Volkmann, a relation $\perp\subset G^2$ is called *orthogonality* if it satisfies the following two conditions:

(O) $0\perp 0$; and from $x\perp y$ the relations $-x\perp -y$, $\frac{x}{2}\perp\frac{y}{2}$ follow.

(P) If an orthogonally additive function from G to an Abelian group is odd, then it is additive; if it is even, then it is quadratic.

Under appropriate conditions on the abelian groups G and H and a subgroup K of H we prove that a function $f: G\to H$ continuous at a point satisfies

$$f(x+y)-f(x)-f(y)\in K\quad\text{for } x,y\in G \text{ such that } x\perp y$$

if and only if there exist a unique continuous additive function $a: G\to H$ and a unique continuous biadditive and symmetric function $b: G\times G\to H$ such that

$$f(x)-b(x,x)-a(x)\in K\quad\text{for } x\in G$$

and

$$b(x,y)=0\quad\text{for } x,y\in G \text{ such that } x\perp y.$$

2. Problems and Remarks

1. Remark *(Solution of the problem by Zoltán Boros)* The following theorem gives a negative answer to the question posed by Z. Boros.

Theorem. *There exists an additive function $f:\mathbb{R}\to\mathbb{R}$ with the properties*

(1) *f is not the sum of a derivation and a continuous additive function*

and

(2) *the set $\{(x, x^{-1}f(x)+xf(x^{-1})) : x\in]0,+\infty[\}$ is not everywhere dense in $]0,+\infty[\times\mathbb{R}$.*

Proof. Let $d:\mathbb{R}\to\mathbb{R}$ be a nonzero derivation and

$$f(x)=d(d(x)),\quad x\in\mathbb{R}.$$

Then it is obvious that f is additive. If f were the sum of a real derivation D and a continuous additive function, i.e., if f were of the form

$$f(x)=D(x)+cx,\quad x\in\mathbb{R},$$

with some real derivation D and $c\in\mathbb{R}$ then, because of $f(1)=D(1)=0$, $c=0$ would follow, i.e. f would be a derivation itself. However this is impossible since the identity

$$f(x^2)=2f(x)+d(x)^2,\quad x\in\mathbb{R},$$

would imply that $d\equiv 0$. Therefore f satisfies (1). On the other hand, for all $x\in]0,+\infty[$, we have that

$$\begin{aligned}
x^{-1}f(x)+xf(x^{-1}) &= x^{-1}d(d(x))+xd(d(x^{-1}))\\
&= x^{-1}d(d(x))+xd(-x^{-2}d(x))\\
&= x^{-1}d(d(x))+x[-x^{-2}d(d(x))+d(-x^{-2})d(x)]\\
&= -xd(x^{-2})d(x)\\
&= 2x^{-2}d(x)^2\geq 0.
\end{aligned}$$

This shows that (2) is satisfied, as well. □

Gyula Maksa

2. Problem *(Stability of power means)* Given a sequence $M_n : \mathbb{R}_+^n \to \mathbb{R}_+$ of symmetric means, we say that the mean $M = (M_n)$ is *ε-multiplicative* (where $\varepsilon > 0$) if

$$\varepsilon \le \frac{M_{nm}(x * y)}{M_n(x)M_m(y)} \le \frac{1}{\varepsilon}, \quad x \in \mathbb{R}_+^n,\, y \in \mathbb{R}_+^m,\, n, m \in \mathbb{N},$$

where, for $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_m)$, the product $x * y$ is defined by

$$x * y = (x_1y_1, \dots, x_ny_1, \dots, x_1y_m, \dots, x_ny_m).$$

If M is 1-multiplicative, then it is simply called multiplicative. It is easy to see that multiplicative means are automatically homogeneous. Therefore, multiplicative quasi-arithmetic means are equal to a power mean.

Assume that $M = (M_n)$ is a quasi-arithmetic mean which is *ε-multiplicative* for some $\varepsilon \ge 0$. Does that imply that then M is close to a power (or Hölder) mean, i.e., there exists an exponent $p \in \mathbb{R}$ such that

$$\varepsilon \le \frac{H_{p,n}(x)}{M_n(x)} \le \frac{1}{\varepsilon}, \quad x \in \mathbb{R}_+^n,\, n \in \mathbb{N}?$$

Zsolt Páles

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