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Report of Meeting

The Seventh Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities January 31 – February 2, 2007 Będlewo, Poland

The Seventh Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities was held from January 31 to February 2, 2007, at the Mathematical Research and Conference Center of Polish Academy of Sciences, Będlewo, Poland.

24 participants came from the Silesian University of Katowice (Poland) and the University of Debrecen (Hungary) at 12 from each of both cities.

Professor Roman Ger opened the Seminar and welcomed the participants to Będlewo.

The scientific talks presented at the Seminar focused on the following topics: equations in a single and several variables, iteration theory, equations on algebraic structures, conditional equations, Hyers–Ulam stability, functional inequalities and mean values. Interesting discussions were generated by the talks.

There was a very profitable Problem Session.

The social program consisted of visiting the castle in Rydzyna and palaces in Rogalin and Pawłowice. During the walk in the park surrounding the Rogalin castle the participants admired three famous oaks: Lech, Czech and Rus. After the excursion they travelled to Poznań were a festive dinner took place in the restaurant “Pod koziołkami” located at the market square of Poznań.

The closing address was given by Professor Zsolt Páles. His invitation to hold the Eigth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities in February 2008 in Hungary was gratefully accepted.

Summaries of the talks in alphabetic order of the authors follow in section 1, problems and remarks in section 2, and the list of participants in the final section.

1. Abstracts of talks

Roman Badora: *Invariant means and the Hahn–Banach theorem*

Under the assumption that every commutative group is amenable we prove the classical Hahn–Banach theorem. Moreover, using the presented method we prove the following version of the Hahn–Banach theorem for groups.

Theorem. *Let H be a subgroup of a commutative group G, $\mathcal{J}$ be a proper linearly invariant ideal of subsets of G, $p\colon G \to \mathbb{R}$ satisfies*

$$p(x+y) \le p(x) + p(y), \quad \Omega(\mathcal{J}) - a.e. \text{ on } G \times G$$

and let $a\colon H \to \mathbb{R}$ be an additive functional fulfilling

$$a(x) \le p(x), \quad \mathcal{J} - a.e. \text{ on } H.$$

Then there exists an additive function $A\colon G \to \mathbb{R}$ such that

$$A(x) = a(x), \quad \mathcal{J} - a.e. \text{ on } H$$

and

$$A(x) \le p(x), \quad \mathcal{J} - a.e. \text{ on } G.$$

Szabolcs Baják: *On a Matkowski–Sutô type equation*
(Joint work with Zsolt Páles)

We deal with the following equation, which is a generalization of the Matkowski–Sutô problem:

$$(\varphi_1+\varphi_2)^{-1}\big(\varphi_1(x)+\varphi_2(y)\big) + (\psi_1+\psi_2)^{-1}\big(\psi_1(x)+\psi_2(y)\big) = x+y,$$

where φ_1, φ_2, ψ_1, ψ_2 are monotonically increasing, continuous functions on the same interval and we assume that each function is four times continuously differentiable. First we establish the connection between φ_1 and φ_2, and ψ_1

and ψ_2 by comparing the derivatives up to the fourth order and taking $x = y$. Then we give the general solutions.

Karol Baron: *Continuity of solutions of the translation equation* (Joint work with Wojciech Chojnacki and Witold Jarczyk)

Assuming that X is a metric space and $F\colon (0,\infty) \times X \to X$ satisfies

$$F(s+t,x) = F(t, F(s,x))$$

for $s,t \in (0,\infty)$ and $x \in X$, we show that:
1. If F is separately continuous, then it is continuous.
2. If X is separable and F is Carathéodory, then F is continuous.

We also show that if X is merely a topological, possibly separable, space, then a separately continuous mapping $F\colon (0,\infty) \times X \to X$ satisfying the above translation equation may fail to be continuous.

Mihály Bessenyei: *Some geometric properties of Beckenbach structures with applications on Hermite–Hadamard-type inequalities*

Beckenbach structures, or as they are also termed, Beckenbach families have the property that prescribing certain points on the plain (with pairwise distinct first coordinates) there exists precisely one member of the family that interpolates the points. Applying Beckenbach families, the classical convexity notion can be considerably generalized: the obtained convexity notion involves the convexity notion induced by Chebyshev systems if the underlying Beckenbach family has a linear structure.

The aim of the talk is to present the basic geometry of Beckenbach lines and give various support properties for generalized convex functions of Beckenbach sense. As a direct consequence of the support properties, Hermite–Hadamard-type inequalities are also obtained.

Zoltán Boros: *Conditional inequalities for additive functions*

Let $H = \{\,(x,y) \in \mathbb{R}^2 \mid xy = 1\,\}$ and $S = \{\,(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\,\}$. The following characterization of derivations is proved.

Theorem. *Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ is an additive function and $K \in \mathbb{R}$ such that either*

$$|\, yf(x) + xf(y)\,| \le K \quad \textit{for all} \quad (x,y) \in H$$

or

$$|\, xf(x) + yf(y)\,| \le K \quad \textit{for all} \quad (x, y) \in S.$$

Then $F(x) = f(x) - f(1)x$, $x \in \mathbb{R}$, *is a derivation.*

Pál Burai: *Matkowski–Sutô equation on symmetrized weighted quasi-arithmetic means*

We solve the invariance equation in the class symmetrized weighted quasi-arithmetic means under mild regularity assumption.

More detailed: Let I a nonempty open interval. A mean M is called symmetrized weighted quasi-arithmetic mean on I if there exists φ a strictly monotone continuous function on I and $\alpha \in]0, 1[$, such that

$$M(x, y) = A^*_\varphi(x, y; \alpha),$$

where

$$A^*_\varphi(x, y; \alpha) = \frac{\varphi^{-1}\left(\alpha\varphi(x) + (1-\alpha)\varphi(y)\right) + \varphi^{-1}\left((1-\alpha)\varphi(x) + \alpha\varphi(y)\right)}{2}.$$

In this case the invariance equation has the following form:

$$A^*_\chi\Big(A^*_\varphi(x, y; \alpha), A^*_\psi(x, y; \beta); \gamma\Big) = A^*_\chi(x, y; \gamma)\,,$$

where $\alpha, \beta, \gamma \in]0, 1[$ and χ, φ, ψ are continuous and strictly monotone functions on I.

We assume that $\chi(x) = x$ and φ and ψ are four times continuously differentiable.

Włodzimierz Fechner: *Functional inequalities connected with the exponential function*

We continue and develop some of the results of Claudi Alsina and Roman Ger published in [1] and connected with the functional inequality

$$f\left(\frac{x+y}{2}\right) \le \frac{f(y) - f(x)}{y - x}, \quad x, y \in I,\ x < y \tag{1}$$

(I is an open real interval). We present the following two results:

- if $f\colon I \to \mathbb{R}$ satisfies (1) and the condition

$$\limsup_{h\to 0+} f(x+h) \geq f(x), \quad x \in I,$$

then f is of the form $f(y) = i(y)e^y$ for $y \in I$, where i is a nondecreasing mapping;
- if $f\colon I \to \mathbb{R}$ satisfies (1) and is continuous then it satisfies the following functional-integral inequality:

$$\frac{1}{y-x}\int_x^y f(t)dt \leq \frac{f(y)-f(x)}{y-x}, \quad x, y \in I,\, x < y. \tag{2}$$

Next, we provide a description of solutions of the inequality (2) and discuss relations between (1), (2) and the following functional-differential inequality:

$$f(x) \leq f'(x), \quad x \in I. \tag{3}$$

Reference

[1] Alsina C., Ger R., *On some inequalities and stability results related to the exponential function*, J. Inequal. Appl. **2** (1998), 373–380.

Roman Ger: *Mean values for vector valued functions on the reals*

Although, in general, a straightforward generalization of the Lagrange mean value theorem for vector valued mappings fails to hold we will look for what can be salvaged in that situation. In particular, we deal with Sanderson's and McLeod's type results of that kind (see [3] and [2], respectively). Moreover, we examine mappings with a prescribed intermediate point in the spirit of the celebrated Aczél's theorem characterizing polynomials of degree at most 2 (cf. [1]).

References

[1] Aczél J., *A mean value property of the derivative of quadratic polynomials—without mean values and derivatives*, Math. Magazine **58** (1985), 42–45.
[2] McLeod R., *Mean value theorems for vector valued functions*, Proc. Edin. Math. Soc. **14** (1965), 197–209.
[3] Sanderson J.D.E., *A versatile vector mean value theorem*, Amer. Math. Monthly **79** (1972), 381–383.

Attila Gilányi: *On subquadratic functions*

In this talk, related to the theory of convex and subadditive functions, we investigate subquadratic mappings, that is, solutions of the functional inequality

$$f(x+y)+f(x-y)\le 2f(x)+2f(y)$$

for real valued functions defined on a space $\mathbb{R}^n$. Especially, we study the lower and upper hulls of such functions and we prove Bernstein–Doetsch-type theorems for them.

Eszter Gselmann: *Dimensions of Cantor-type sets*

In this talk we determine the most important properties of so-called Cantor-type sets. Let Λ denote the set of all those strictly decreasing sequences (λ_n) of positive real numbers for which $\sum_{n=1}^{\infty}\lambda_n = L < +\infty$. Moreover, for $(\lambda_n)\in\Lambda$, let

$$C\left((\lambda_n)\right)=\left\{x\in[0,L] \mid \text{ exists } (\varepsilon_n):\mathbb{N}\to\{0,1\}\,,\text{ that } x=\sum_{n=1}^{\infty}\varepsilon_n\lambda_n\right\}.$$

If the sequence $(\lambda_n)\in\Lambda$ is interval filling, then $C\left((\lambda_n)\right)=[0,L]$. Therefore we will deal with the case of non interval filling sequences. We will show that the set $C\left((\lambda_n)\right)$ is continuum, compact, does not contain any interval and has zero Lebesgue measure, if we assume that the sequence $(\lambda_n)\in\Lambda$ is a Cantor-type sequence. Finally, we determine the dimensions of Cantor-type sets, as well.

Attila Házy: *On the stability of $\boldsymbol{t}$-convex functions*

A real valued function f defined on D (where D is an open convex set of a normed space X) is called (d,t)-convex if it satisfies the inequality

$$f\left(tx+(1-t)y\right)\le tf(x)+(1-t)f(y)+d(x,y)$$

for all $x,y\in D$, where $d\colon X\times X\to\mathbb{R}$ is a given ψ-subhomogeneous translation invariant deviation and $t\in]0,1[$ is a fixed parameter.

The main result of our talk states that if f is locally bounded from above at a point of D and (d,t)-convex, then it satisfies the following convexity-type inequality (under some assumptions)

$$f\big(sx+(1-s)y\big)\le sf(x)+(1-s)f(y)+\varphi(s)d(x,y)$$

for all $x, y \in D$ and $s \in [0,1]$, where $\varphi\colon [0,1] \to \mathbb{R}$ is defined as the fixed point of a certain contraction. This result offers a generalization of the celebrated Bernstein and Doetsch theorem and the recent results by Ng and Nikodem, Páles and the author.

Barbara Koclęga-Kulpa: *Some functional equations characterizing polynomials*
(Joint work with Tomasz Szostok and Szymon Wąsowicz)

We present a method of solving functional equations of the type

$$F(x) - F(y) = (x-y)[b_1 f(\alpha_1 x + \beta_1 y) + \cdots + b_n f(\alpha_n x + \beta_n y)],$$

where $f, F\colon P \to P$ are uknown functions acting on an integral domain P and parameteres $b_1, \ldots, b_n,\ \alpha_1, \ldots, \alpha_n,\ \beta_1, \ldots, \beta_n \in P$ are given. We prove that under some assumptions on the parameters involved, all solutions of such kind of equations are polynomials. We use this method to solve some concrete equations of this type. For example, the equation

$$(1)\quad 8[F(y) - F(x)] = (y-x)\left[f(x) + 3f\left(\frac{x+2y}{3}\right) + 3f\left(\frac{2x+y}{3}\right) + f(y)\right]$$

for $f\colon \mathbb{R} \to \mathbb{R}$ is solved without any regularity assumptions. It is worth noting that (1) is a well known quadrature rule used in numerical analysis.

Rezső L. Lovas: *On a functional equation involving means*
(Joint work with K. Lajkó, Gy. Maksa and Zs. Páles)

In my talk I shall consider the functional equation

$$f(px + (1-p)y) + f((1-p)x + py) = f(x) + f(y), \quad x, y \in I,$$

where $0 < p < 1$ is a fixed parameter, and $f\colon I \to \mathbb{R}$ is an unknown function. I shall completely characterize the equivalence of this equation and Jensen's functional equation in terms of the algebraic properties of the parameter p.

Grażyna Łydzińska: *On some set-valued iteration semigroups*

Let X be an arbitrary set. We present the necessary and sufficient conditions for a set-valued function $A\colon X \to 2^{\mathbb{R}}$ under which a family of multifunctions of form $A^{-1}\big(A(x) + \min\{t, q - \inf A(x)\}\big)$, where $q\colon = \sup A(X)$, is an iteration semigroup.

Gyula Maksa: *Stability of a functional equation of information theory*

In this talk we present the following

Theorem. *Let* $D = \{(x, y) \in \mathbb{R}^2 : x, y \in [0, 1[,\ x + y \leq 1\}$, $\alpha, \varepsilon \in \mathbb{R}$, $0 < \alpha \neq 1$, $0 \leq \varepsilon$, *and* $f\colon [0, 1] \to \mathbb{R}$. *Suppose that*

$$\left| f(x) + (1-x)^{\alpha} f\left(\frac{y}{1-x}\right) - f(y) - (1-y)^{\alpha} f\left(\frac{x}{1-y}\right) \right| \leq \varepsilon$$

for all $(x, y) \in D$. *Then there exists a function* $g\colon [0, 1] \to \mathbb{R}$ *such that*

$$g(x) + (1-x)^{\alpha} g\left(\frac{y}{1-x}\right) = g(y) + (1-y)^{\alpha} g\left(\frac{x}{1-y}\right)$$

for all $(x, y) \in D$ *and*

$$|f(x) - g(x)| \leq 7|2^{1-\alpha} - 1|^{-1}\varepsilon$$

for all $x \in [0, 1]$.

Janusz Matkowski: *On some generalizations of the Lagrange and Cauchy mean-value theorems*

Some generalizations of the classical mean-value theorems of Lagrange and Cauchy, involving arbitrary means, will be presented. The families of means corresponding to the families of Lagrange and Cauchy means will be considered.

Fruzsina Mészáros: *Further generalizations in connection with an old problem of J. Aczél*
(Joint work with Zs. Ádám, K. Lajkó, and Gy. Maksa)

Let G be an arbitrary group written additively. We give the general solution of the functional equation

$$f(x)\, f(x+y) = f(y)^2\, f(x-y)^2\, g(y), \quad x, y \in G, \tag{1}$$

where $f, g\colon G \to \mathbb{R}$ are unknown functions. Moreover we consider a pexiderized equation which is connected with equation (1) and the sequenced problems in [4].

References

[1] Aczél J., *Some general methods in the theory of functional equations in one variable, new applications of functional equations*, Uspechi Mat. Nauk **11** (1956), no. 3(69), 3–68 (in Russian).
[2] Aczél J., *Some general methods in the theory of functional equations in one variable and new applications of functional equations*, *MTA III.* Oszt. közl. **9** (1959), 375–422 (in Hungarian).
[3] Aczél J., *Lectures on functional equations and their application*, In: Mathematics in Science and Engineering, Vol. 19, Academic Press, New York–London 1966.
[4] Ádám Zs., Lajkó K., Maksa Gy., Mészáros F., *Sequenced problems for functional equations*, Teach. Math. and Comp. Sci. **4(1)** (2006), 179–192.
[5] Ádám Zs., Lajkó K., Maksa Gy., Mészáros F., *Functional equations on group*, Ann. Math. Sil. (in press).

Janusz Morawiec: *On continuous and bounded solutions of some functional equations*
(Joint work with Rafał Kapica)

Let $(\Omega, \mathcal{A}, P)$ be a probability space, X be a complete separable metric space, and $f\colon X \times \Omega \to X$ be a random-valued function. We consider continuous and bounded solutions $\varphi\colon X \to \mathbb{R}$ of the equations

$$\varphi(x) = \int_\Omega \varphi(f(x,\omega))dP(\omega), \qquad \varphi(x) = 1 - \int_\Omega \varphi(f(x,\omega))dP(\omega).$$

Andrzej Olbryś: *The support theorem for t-Wright convex functions and its consequences.*

Let $t \in (0,1)$ be a fixed number, $L(t)$ be the smallest field containing the set $\{t\}$, and let X be a linear space over the field K, where $L(t) \subset K \subset \mathbb{R}$. Assume, moreover that $D \subset X$ is a $L(t)$-convex set, i.e., $\alpha D + (1-\alpha)D \subset D$, for all $\alpha \in L(t) \cap (0,1)$.

A function $f\colon D \to \mathbb{R}$ is said to be a t-Wright convex iff

$$(1) \qquad f(tx + (1-t)y) + f((1-t)x + ty) \le f(x) + f(y), \quad x, y \in D,$$

it is said to be a t-Wright affine if the above inequality is satisfied with equality. If (1) is fulfilled for $t = \frac{1}{2}$ then f is called convex in the sense of Jensen. In the case where $K = \mathbb{R}$ and D is a convex set we say that the function $f\colon D \to \mathbb{R}$ is a Wright convex if the inequality (1) is fulfilled for all $\lambda \in [0,1]$ and for all $x, y \in D$.

We present a theorem giving necessary and sufficient conditions that for an arbitrary point $y \in D$ and a t-Wright convex function $f\colon D \to \mathbb{R}$ there exists a support function $a_y\colon D \to \mathbb{R}$, i.e., the function satisfying the following conditions:

(i) $a_y(tx+(1-t)z)+a_y((1-t)x+tz)=a_y(x)+a_y(z), \quad x,z\in D,$
(ii) $a_y(x)\le f(x), \quad x\in D,$
(iii) $a_y(y)=f(y).$

As a consequences of this theorem we obtain a characterization of some subclasses of the class of all t-Wright convex functions. In particular, we give necessary and sufficient conditions under which a t-Wright convex function is Jensen-convex, Wright-convex or has the representation $f=F+A$, where F is a Jensen-convex and A is a t-Wright-affine function.

Zsolt Páles: *On a linear functional equation*

Given a nonzero signed Borel measure μ on the interval $[0,1]$, we consider the functional equation

$$\int_0^1 f\big((1-t)x+ty\big)d\mu(t)=0, \quad x,y\in I, \tag{1}$$

where $f\colon I\to\mathbb{R}$ is an unknown continuous function defined on the open interval I.

In the particular case $\mu=\sum_{i=0}^{n}(-1)^n\binom{n}{i}\delta_{\frac{i}{n}}$ (where δ_t stands for the Dirac measure concentrated at a point $t\in[0,1]$), equation (1) can be rewritten as

$$\sum_{i=0}^{n}(-1)^n\binom{n}{i}f\left(x+\frac{i}{n}(y-x)\right)=0, \quad x,y\in I, \tag{2}$$

which is the well-known Fréchet functional equation.

Similarly to what holds for the continuous solutions of the Fréchet equation, we show that the continuous solutions of (1) are polynomials whose degree is determined by the moments of the measure μ.

Maciej Sablik: *On a problem of P.K. Sahoo*
(Joint work with Arkadiusz Lisak)

During the 44th International Symposium on Functional Equations held in Louisville, Kentucky, USA, in May 2006, P.K. Sahoo presented results concerning the following equation

$$g(y)-h(x)=(y-x)[f(x)+2k(sx+ty)+2k(tx+sy)+f(y)], \tag{1}$$

which stems from trapezoidal rule of computing an integral. The general solution has been given in the class of functions f,g,h,k mapping $\mathbb{R}$ into $\mathbb{R}$ and such that g and f are twice differentiable, and k is four times differentiable. Our aim is to relax the regularity assumptions, and, in the case where s,t are rational, to solve the equation with no regularity assumption at all. We also raise the question of the reason for which discontinuous functions appear on the list of solutions.

References

[1] Sablik M., *Remark 10.* Remark at the *44th ISFE*, Louisville, Kentucky, USA, May 14–20, 2006.
[2] Sahoo P.K.,*On a functional equation associated with the trapezoidal rule.* Talk at the *44th ISFE*, Louisville, Kentucky, USA, May 14–20, 2006.
[3] Sahoo P.K., *Remark 14.* Remark at the *44th ISFE*, Louisville, Kentucky, USA, May 14–20, 2006.

Dariusz Sokołowski: *On the asymptotic behaviour of integrable solutions to a linear functional equation*

We investigate the equation

$$\varphi(x) = \int_S \varphi(x + M(s))\sigma(ds), \tag{1}$$

where (S, Σ, σ) is a measure space with a finite measure σ and $M\colon S \to \mathbb{R}$ is a Σ-measurable bounded function with $\sigma(M \neq 0) > 0$. By a *solution* of (1) we mean a Borel measurable real function φ defined on an interval of the form $(a, +\infty)$, such that for every $x > a + \sup\{|M(s)| : \ s \in S\}$ the integral occuring in (1) exists and (1) holds.

We show that if $\sigma(S) \neq 1$, then there are positive reals α, β such that for any positive solution φ of (1) which is integrable on a vicinity of infinity

$$\liminf_{x\to+\infty} \varphi(x)e^{\alpha x} = 0 \qquad \text{and} \qquad \limsup_{x\to+\infty} \varphi(x)e^{\beta x} = +\infty.$$

Tomasz Szostok: *On some Cauchy-type equation*
(Joint work with Michał Baczyński)

Let $r_1, r_2 > 0$ be given number and let $f\colon [0, r_1] \to [0, r_2]$ be a function. Recently the equation

$$f(\min(x + y, r_1)) = \min(f(x) + f(y), r_2) \tag{1}$$

was considered by Michał Baczyński in connections with some applications in fuzzy logic. We observe that functions $x \to \min(x, r_1)$ and $x \to \min(x, r_2)$

ocurring in this equation may be replaced by any functions acting on suitable intervals. Therefore we consider the equation.

$$f(m_1(x+y)) = m_2(f(x)+f(y)), \tag{2}$$

where $f\colon [0,r_1] \to [0,r_2]$, $m_1\colon [0,2r_1] \to [0,r_1]$, and $m_2\colon [0,2r_2] \to [0,r_2]$ are unknown. It is interesting to observe that this equation is a joint generalization of (1) and of the Jensen equation. We present a solution of (2) under some assumptions on m_1 and m_2.

Adrienn Varga: *A functional equation involving four weighted arithmetic means*

In this talk we study the functional equation

$$f(\alpha x+(1-\alpha)y) + f((1-\alpha)x+\alpha y) = f(\beta x+(1-\beta)y) + f((1-\beta)x+\beta y),$$

which holds for all $x, y \in I$ where $I \subseteq \mathbb{R}$ is a non-void open interval, $f\colon I \to \mathbb{R}$ is an unknown function, $0<\alpha<1$, $\alpha \neq \frac{1}{2}$, $0<\beta<1$, and $\alpha \notin \{\beta, 1-\beta\}$. The key observation is that the solutions are exactly the p-Wright affine functions, where $p = \frac{\alpha+\beta-1}{2\alpha-1}$. They have the form

$$f(x) = A_2(x,x) + A_1(x) + A_0, \quad x \in I,$$

with some $A_k\colon \mathbb{R}^k \to \mathbb{R}$ k-additive functions ($k=0,1,2$), A_2 is symmetric and $A_2(px,(1-p)x) = 0$ for all $x \in \mathbb{R}$. If A_2 is identically zero then f is Jensen affine. We are interested in the solutions with nontrivial biadditive part. As we show, the existence of such a solution depends on the algebraic properties of the parameter p.

2. Problems and Remarks

1. Problem. Let $\mathcal{A}$ denote the set of all additive functions $f\colon \mathbb{R} \to \mathbb{R}$ and let $\mathcal{D}$ denote the set of elements $f \in \mathcal{A}$ of the form

$$f(x) = d(X) + cx, \quad x \in \mathbb{R},$$

where $c \in \mathbb{R}$ and d is a derivation.

For $f \in \mathcal{A}$, let

$$\Phi[f](x) = \frac{1}{x} f(x) + x f\left(\frac{1}{x}\right), \quad x \in \mathbb{R}^+ = (0, \infty).$$

Among others, the following result is presented in my talk

Theorem. *If $f \in \mathcal{A}$ is such that $\Phi[f]$ is bounded on a non-void interval $I \subset \mathbb{R}^+$ then $f \in \mathcal{D}$.*

2. Problem. Is it true that, for every $f \in \mathcal{A} \setminus D$, $\Phi[f]$ is dense in $\mathbb{R}^+ \times \mathbb{R}$?

Z. Boros

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