Annales Mathematicae Silesianae 16 (2003)

Prace Naukowe Uniwersytetu Śląskiego nr 2142, Katowice

Problem session

The 4th Czech and Polish Conference on Number Theory Cieszyn, June 11–14, 2002

The Problem Session, chaired by Andrzej Schinzel, took place on June 14, 2002. The following problems were proposed.

WLADYSLAW NARKIEWICZ:

Find an algorithm for solving the equation

$$u+v+w=1$$

in units of an algebraic number field.

ANDRZEJ SCHINZEL:

(Unpublished problem of T. Ordowski)

In 1944 Chowla and Mian considered the sequence

$$1, 2, 4, 8, 13, \ldots,$$

where a_n is the smallest natural number with the property that the set of differences $a_n - a_i$ for $1 \leqslant i < n$ is disjoint from the set of all differences $a_j - a_i$ for $1 \leqslant i < j < n$. The first question is to find an estimate for the n–th term of the sequence better than the known estimate

$$\frac{1}{2}n^2 + O(n) \leqslant a_n \leqslant \frac{1}{6}n^3 + O(n^2).$$

The second question is to decide whether or not the series

$$\sum_{n=1}^{\infty} \frac{1}{a_{n+1} - a_n}$$

converges.

REFERENCES

[1] Mian, Abdul Majid, and S. Chowla, *On the B_2 sequences of Sidon*, Proc. Nat. Indian Acad. Sci. India. Sec. A. 14 (1944), 3–4.

Collected and edited by K. Szymiczek

KAZIMIERZ SZYMICZEK:

Let K be a number field and $\mathcal{O}_K$ its maximal order, that is, the ring of all algebraic integers in K. Let $\mathcal{O}$ be any order of K (that is, a subring of the maximal order containing a basis for K over the rationals). Find the kernel and the cokernel of the natural ring homomorphism

$$W(\mathcal{O}) \to W(\mathcal{O}_K),$$

where, for a commutative ring R, $W(R)$ is the Witt ring of bilinear spaces over R.

ANDRZEJ ROTKIEWICZ:

Do there exist infinitely many Dickson–Fibonacci pseudoprimes not divisible by 5 which are not Frobenius–Fibonacci pseudoprimes?

The least such pseudoprime is $2737 = 7 \cdot 13 \cdot 23$.

ANDRZEJ ROTKIEWICZ:

Do there exist infinitely many Fibonacci pseudoprimes of the second kind which are not Frobenius–Fibonacci pseudoprimes?

The least such pseudoprime is $6479 = 11 \cdot 19 \cdot 31$.

REFERENCE

[1] A. Rotkiewicz, *Lucas and Frobenius pseudoprimes*, Ann. Math. Sil. (to appear).

ANDRZEJ SŁADEK:

Let

$$3 \prec 5 \prec 7 \prec 9 \prec \ldots \prec 2\cdot 3 \prec 2\cdot 5 \prec \ldots \prec 2^2\cdot 3 \prec 2^2\cdot 5 \prec \ldots \prec 2^3 \prec 2^2 \prec 2 \prec 1$$

be the Sharkovski's ordering of the set of natural numbers $\mathbb{N}$. For a function $f : I \longrightarrow I$, where $I = [a, b] \subset \mathbb{R}$ or $I = \mathbb{R}$, Sharkovski proved in 1963 the following theorem: If f is continuous, then

$$n \in Cycl(f) \Longrightarrow \underset{m}{\forall} \{n \prec m \Rightarrow m \in Cycl(f)\}.$$

Here $Cycl(f) := \{n \in \mathbb{N}; f \text{ has an } n-\text{cycle}\}$.

It is known that for any $n \in \mathbb{N}$ there exists a continuous function $f : I \longrightarrow I$ such that $Cycl(f) = \{m \in \mathbb{N}; n \preceq m\}$. The functions constructed in the literature are piecewise polynomial.

The question is whether the function f can actually be taken as a polynomial, that is, we ask if the following statement holds true:

$$\forall_{n\in\mathbb{N}} \ \exists_{f\in\mathbb{R}[X]} \ Cycl(f) = \{m \in \mathbb{N}; n \preceq m\}.$$

JÁNOS TÓTH:

Let $F(n)$ be the number of solutions $(x_1, x_2, \ldots, x_n) \in \mathbb{N}^n$ of the diophantine equation

$$x_1 x_2 \cdots x_n = n(x_1 + x_2 + \cdots + x_n)$$

such that $x_1 \leqslant x_2 \leqslant \cdots \leqslant x_n$. It is known that

$$\limsup_{n\to\infty} F(n) = \infty \quad \text{and} \quad F(n) = O(n^2).$$

The question is whether

$$\lim_{n\to\infty} F(n) = \infty.$$

Moreover, do there exist positive constants c and α so that

$$\lim_{n\to\infty} \frac{F(n)}{cn^\alpha} = 1 \ ?$$

REFERENCE

[1] J. Bukor, P. Filakovszky, J. Tóth, *On the diophantine equation* $x_1x_2\cdots x_n = h(n)(x_1 + x_2 + \cdots + x_n)$, Ann. Math. Sil. 12 (1998), 123–130.

JAN KREMPA:

A subset $\mathcal{S} \subset \mathbb{N}$ is said to be *Pythagorean* if for any $n \in \mathbb{N}$ and for any distinct elements $s_1, \ldots, s_n \in \mathcal{S}$ there exists $t \in \mathbb{N}$ such that

$$(1) \qquad \sum_{i=1}^{n} s_i^2 = t^2.$$

Any singleton is a Pythagorean set, all 2-element Pythagorean sets are well known, and it is an open question if there exists a 3-element Pythagorean set. It can be checked that any Pythagorean set is finite. This suggests the following question. Does there exist $k \in \mathbb{N}$ such that any Pythagorean set has at most k elements?

A Pythagorean set $\mathcal{S}$ is said to be *primitive* if there is no nontrivial common divisor for all elements of $\mathcal{S}$. A primitive Pythagorean set contains only one odd number. Let $\mathcal{S}$ be a primitive Pythagorean set with odd element $s \in \mathcal{S}$. What is the exact upper bound for the cardinality of $\mathcal{S}$ as a function of s?