GYŐRGY TARGONSKI'S LIFE AND WORK

BOGDAN CHOCZEWSKI AND JOLÁN TARGONSKI

Abstract. The paper is based on the Curriculum Vitae of György Targonski (March 27. 1928 – January 10. 1998) written by the second author, and is completed by a survey of his domains of research with special attention to selected papers. Professor Targonski's scientific legacy consists of over hundred research papers, books, lecture notes and reports, contributions to meetings (problems, remarks, abstracts of lectures) and of proceedings of the ECIT. These are all documented in the three lists attached to the article.

At the end of 1997 we fully expected to celebrate the 70th birthday of Professor György Targonski at the forthcoming European Conference on Iteration Theory (ECIT 98) in Muszyna-Złockie. Although we were aware of his precarious state of health, we knew that he had attended the 6^{th} International Conference on Functional Equations and Inequalities (ICFEI, also in Muszyna-Złockie) in June 1997, had given a talk at the 36^{th} International Symposium on Functional Equations (ISFE) in Graz, Austria, in September, had discussed the organization of ECIT 98 on both meetings, and had sent out the invitations to ECIT 98 in November.

Professor Targonski never complained about his health. Despite the effects of the strokes he sufferred in 1981 and 1992, he always, when asked how he was feeling, replied: *"Good, thanks to God"*. He had always wished to be conscious when he died, and this wish, at least, was fulfilled by God: He was conscious when he received the Last Sacrament. Ten minutes later,

Received: December 5, 1998.

AMS (1991) subject classification: Primary 01A70, 39B12, 47B38, 81U99.

he closed his eyes and died quietly, at noon on 10^{th} of January, 1998. He is buried in the cemetary called the Neuer Südfriedhof in Munich, Germany.

Wanting him to be among us in spirit, in the way he will always remain in our hearts and thoughts, we have dedicated the ECIT 98 to his memory.

Childhood – Berlin (1928–1938)

György Iván Targonski was born on the 27th of March, 1928, in Budapest. He was the only child of Anton Targoński and Rózsa Simonovits. His father was Polish, his mother Hungarian. They lived in Berlin. His father did not speak Hungarian, so the parents spoke to each other in French, but with the child they spoke German. This was disturbing to the little boy. He often said: *that was the reason why I never could learn French properly.*

When he was eight years old, he lost his father. *From that time on*, as he often said, *I was the head of the family.* He promised his dying father to take care of his mother as long as she lived. He kept his promise. Although she lived in Geneva and he in Frankfurt, he took care of his mother in every way. He phoned her every day, and visited her very frequently in Geneva even at the time when his health was already very poor. As a good child, he was always with her on each of her birthdays, and on each of the bigger holidays. His mother died in 1992 at the age of one hundred.

Youth – Budapest (1938–1956)

In 1938 Mrs. Rózsa Targoński returned with György to Budapest. It was only then than he began to learn Hungarian. Forced by a kind of tuberculosis to stay in bed for about ten months, he read all the novels of Jókai, the famous nineteenth-century Hungarian writer. Through that, he learned perfect Hungarian, though a bit old-fashioned.

He started to get interested in mathematics at an early age. He said: *My first thoughts on iteration were at the age of about fifteen. I sat on a bench in the Városmajor park and I decided to investigate the behaviour of functions which result from n-times substitution of $f(x)$ into itself.*

He finished the Verbőczy Gimnázium in 1947 Summa cum Laude, and the Pázmány Péter (the present Eőtvős Loránd) University with a Diploma (equivalent to a Master of Science) in mathematics in 1951.

His first achievement was the paper [I.1] which won a prize in a Hungarian mathematical competition in 1951.

He became an Assistant Lecturer at the Technical University in Budapest in 1952, and in this position completed his thesis [I.7] for the Candidate

of Sciences degree. He presented it in 1955 and Professor Béla Szőkefalvi Nagy was the referee. The thesis was accepted, but due to circumstances connected with the Hungarian Uprising of 1956 the degree was never awarded.

Europe (1956–1963)

When the Hungarian Uprising began on October 23, 1956, György was very optimistic about the freedom which was finally to come to Hungary. On November 4, 1956, Soviet tanks came into Budapest and began firing their cannons at houses and at people on the streets. It soon became clear that freedom had lived only a short time.

On the morning of November 13 he, together with his 66 year-old mother and his fianceé, began an adventureous journey to freedom. They walked to one of the suburban railway stations in Budapest and caught a freight train going to the "West". It was a two-day journey by train and on foot until they reached the Austrian border. On the foggy early morning of November 15, they crossed the Fertő Channel by boat to Austria with the aid of three Hungarian border guards. After crossing Austria they were accepted as refugees by Switzerland .

György and Jolán got married on the 15th of December, in Zürich. In the same month, György went to the Eidgenössische Technische Hohschule (ETH) in Zürich and met Professor Benno Eckmann. Through him, he got an assistantship at the ETH and started to teach there on the next working day.

In March 1957 the Targonskis moved to Cambridge, England, where György started preparing his Ph. D. thesis at the famous Cavendish Laboratory under the supervision, first of Professor D. R. Hartree and then of Professor R. G. Eden, He received his Ph. D. (Cantab) in April 1963 in a very solemn medieval-like ceremony.

To earn some money for living expenses he worked as supervisor in King's College for three years, during which he attended dinner at King's College High Table once a week. He was very impressed by this experience. His subsequent appointments were: Assistant Lecturer at Queen Mary College, London University, 1959 – 60, Attaché de Récherche in the Theoretical Division of the Centre Européen de Récherche Nucléaire (CERN), Geneva, 1960 –61, and Assistant at the Institut de Physique Théorique of the University of Geneva, 1961 – 63.

György Targonski's Ph. D. thesis [I.13] deals with some problems of quantum theory which are connected with the theory of linear operators.

Section 3 of the thesis is purely mathematical. It is observed that scale transformations (i.e. general transformations in the independent variables) often play an important part in quantum physics. We report on two problems considered in this Section of [I.13].

Let X be a normed space and B a Banach algebra of (real or complex valued) functions on X. Moreover let $P' = h(P)$ be a not necessarily linear transformation of the underlying space called a "general scale transformation". The mathematical starting point of this section is the following fact:

The mapping h generates a linear transformation H on B, given by

$$Hf(P) = f(P') = f[h(P)].$$

This substitution operator is not only linear but also multiplicative:

$$H[f_1(P)f_2(P)] = [Hf_1(P)]\,[Hf_2(P)].$$

The question arises whether the converse is also true? It is answered in the affirmative for two special cases:

a) the Banach algebra $C[0, 1]$ of all continuous complex valued functions defined on the unit interval,

b) the Banach algebra A of entire functions with norm $\|f\| = \sum a_n$ where $f = \sum a_n z^n$.

The results a) and b) are characterizations of multiplicative linear operators H acting on the underlying Banach algebra.

Another topic in this Section are B_ξ algebras. These are Banach algebras of functions with the additional property that for $f \in B_\xi$ and real $\xi \geq 0$ the function f^ξ defined by $f^\xi(P) = [f(P)]^\xi$ also belongs to B_ξ. This is to be understood as: f^ξ is a multi valued function which assumes at P all complex values of $f(P)^\xi$.

The location of the eigenvalues of a substitution operator acting on it is determined. The Schröder equation (the eigenvalue equation of the substitution operator H):

$$Hf(P) = f(h(P)) = \lambda f(P)$$

is also discussed as well as the problem of invariants of a substitution operator on B_ξ.

Four other papers related to scattering theory were published in period 1959–63 (in Il Nuovo Cimento: [I.9] and [I.11] (with A. Martin) and in Helvetica Phys. Acta: [I.12] and [I.14], both joint with B. Misra and D. Speiser).

Fordham University – New York (1963–1974)

György Targonski was appointed associate professor at Fordham University from 1963, and full professor from 1966. Fordham University is a private Jesuit University in New York City.

One of Targonski's hobbies was to read Science Fiction (he had a collection of a few hundred titles). At the time when he was applying for positions in the United States, he read a science fiction book about a mathematician who had just returned from space to get a job at Fordham and whose name was Agronski. So he thought he also had to send his application there. He got the position which he always considered as a very interesting chain of coincidences.

His academic years at Fordham were complemented by summers spent at the Forschungsinstitut für Mathematik of the ETH in Zürich. The two sabbatical years of 1965/66 and 1972/73 he also spent as a Visiting Professor at the ETH.

At Fordham Targonski worked mainly on operator theory, especially on Carleman and Bourlet operators. We shall go into a little bit of detail by reviewing the relevant papers [I.20] and [I.21] which were published in 1967. His lecture notes [I.22] *Seminar on Functional Operators and Equations* also appeared in this year.

The main result of [I.20] may be described as follows:

A *Carleman operator* is, roughly speaking, an integral operator K acting on $L^2(a,b)$

$$(Kf)(x) = \int_a^b k(x,y)f(y)dy$$

whose kernel k is not necessarily square integrable (on $(a,b)^2$) but the functions $k(x,\cdot)$ are (on (a,b)), for almost all $x \in (a,b)$.

The *Convergence Theorem* says the following:

Assume we are given two complete, orthonormal systems (φ_n), (ψ_n) in $L^2(a,b)$, the unitary operator U transporting (φ_n) into (ψ_n), and the self-adjoint operator V acting on (φ_n),

$$U\varphi_n = \psi_n, \quad V\varphi_n = \sqrt{c_n}\varphi_n,$$

where $c_n \geq 0$, $\liminf c_n = 0$. Then the implication

$$\sum c_n|\varphi_n(x)|^2 < \infty, \implies \sum c_n|\psi_n(x)|^2 < \infty$$

holds true if, and only if, the composition UV is a Carleman operator.

The other important paper [I.21] deals with the characterization of Bourlet operators.

Let A denote a commutative algebra with unit element e and without divisors of zero and Ω – a linear operator defined on A satisfying a multiplication law

$$\Omega(uv) = F(u, v, \Omega u, \Omega v) \tag{F}$$

where F is an entire complex function in four variables. Operators satisfying an equation (F) are called *Bourlet operators*. Bourlet (1897) stated without proof the following results of which proofs were supplied by Gy. Targonski in [I.21]:

If Ω satisfies (F) then the equation is of one of the following three types:

$$\Omega(uv) = \frac{1}{2}[u\Omega v + v\Omega u] \tag{M}$$

$$\Omega(uv) = u(\Omega v + \lambda v) + v(\Omega u + \lambda u) - \lambda uv \tag{D}$$

$$\Omega(uv) = A(\Omega u - \mu u)(\Omega v - \mu v) + \mu uv \tag{H}$$

where $A \neq 0, \lambda$ and μ are suitable constants. Moreover in case (M) the operator Ω has the following explicit form:

$$\Omega u = \Omega e \cdot u \tag{M*}$$

If P is the algebra of polynomials of one complex variable, with unit element, then the operator Ω has the following representation (with an $\omega \in P$ as a parameter).

$$(\Omega f)(z) = \omega(z)\frac{d}{dz}f(z) - \lambda f(z), \qquad z \in C, \tag{D*}$$

$$(\Omega f)(z) = \frac{1}{A}f[A\omega(z)] + \mu f(z), \qquad z \in C. \tag{H*}$$

[Of course, (M*) now reads: $\Omega f = \omega f$.]

It is also observed that the equations $\Omega f = k$ $(k \in P)$ which are linear differential in case (D*) and linear iterative in case (H*) are then of common origin. [Bourlet operators were discussed again on the ECIT 92 by L. Berg (in the real case) (cf. [III.5], 51-54).]

In this period Győrgy Targonski was also dealing with functional equations, to mention only [I.17], the paper which discusses the relationship

between continuous iteration, Schröder's equation, the translation equation and the generalized Chebyshev systems (P_n) given by

$$P_{n+1}(x) = R(P_n(x), x); \qquad P_1(x) = x,$$

with a suitable "addition formula" R.

What is certainly to be mentioned in this context is Targonski's Problem (1970, cf. [II.12]) on the Pre-Schröder System

$$\text{(PS)} \qquad [\varphi \circ f]^n = (\varphi \circ f^n)[\varphi]^{n-1}, \qquad n \in \mathbb{N},$$

connected with the Schröder Equation

$$\text{(S)} \qquad \varphi \circ f = \lambda\varphi$$

and the later posed problem concerning the Pre-Abel System (1973, cf. [II.18])

$$\text{(PA)} \qquad \varphi \circ f^n = n\varphi \circ f - (n-1)\varphi, \qquad n \in N,$$

connected with the Abel Equation

$$\text{(A)} \qquad \varphi \circ f = \varphi + 1.$$

Targonski asked whether (PS) and (S) (resp. (PA) and (A)) are equivalent. He published two papers on both problems: [I.26] on (PS), joint with Marek Kuczma; and [I.30] on (PA). The problems attracted the attention of several other mathematicians in the decade 1975–1985, cf. the survey article by Professor Zenon Moszner, published in this volume.

Philipps-Universität Marburg (1974–1998)

In June 1972 the Targonskis returned to Europe, to the *good old continent* as Győrgy used to say. He also said very often: *I was born here and I also want to be buried here.*

After a year at the Ruhr-Universität Bochum in 1974, Győrgy Targonski got a position as Full Professor of Applied Mathematics at the Philipps University of Marburg. Since 1993 he was Professor Emeritus at this University. The professorship at Marburg University was a very successful period of his life. It was actually "his" university. He enjoyed teaching, research work, scientific contacts. He was held in high esteem as professor and colleague. He created there a research school in iteration theory.

In 1977 György Targonski became a German citizen. In 1982 he moved to Frankfurt to live in a "big" city as he liked to do (recall Berlin, Budapest, Zürich, London, Geneva, New York), and to be closer to Geneva where his mother was living.

It is impossible to review here, even briefly, Professor Targonski's results in iteration theory and related functional equations that he obtained in this very fruitful 25-year period of his life. However, one cannot omit mention of his book *Topics in Iteration Theory* [I.34]. He himself wrote in 1995 (cf. [I.50]): it is *to the best of my knowledge, the first book on iteration theory in general.*

Let us now quote a few sentences from Professor Abe Sklar's review of the book (published in Bull. (NS) AMS 9 (1983), 345-348):

The treatment of the topics is reminiscent of that in old "Encyklopädie" articles ... however, Targonski often furnishes proofs. And his discussions, though brief, are consistently illuminating ... Perhaps the most valuable feature of the book is the presentation of material that would otherwise be practically inaccessible. This applies to some of the work of the author himself and his students U. Burkart, R. Graw, and G. Zimmermann. While much of the work of this "Marburg school" has been published, some of it has not, or has only appeared in the form of doctoral dissertations ... This also applies to the material on the "Pilgerschritt" transformation in Chapter 4. The concept ... is due to R. Liedl, who introduced it as a method for obtaining (in certain cases) an embedding of a function in a family of generalized iterates by successive approximations ... Professor Targonski has done a great service for all of us interested in iteration theory.

Professor Targonski always wanted to be up-to-date in iteration theory. In 1984 he published the paper [I.36] and in 1995 the expository article [I.50] *Progress in iteration theory since 1981.* The contents of [I.36] shows, among others, the problems in iteration theory which remained unsolved since 1984.

We now choose from [I.50] the problem of "phantom iterates" for a bit more detailed presentation. The problem appears indirectly in Targonski's book [I.34] but it was first proposed under this name at the ECIT 84 (cf.[I.40]).

The idea is the following.

In general, for a given selfmapping f of a set S there is no mapping g on S such that

$$\text{(IR)} \qquad g^r = f, \qquad r \geq 2.$$

To find a kind of iterative root of f let us immerse the semigroup of mappings under composition into a larger semigroup in which (IR) has a solution.

More precisely, let Φ be a family of functions from a set S into a commutative algebra A. Under the operations of A applied "pointwise" the family Φ becomes itself a commutative algebra.

If there is a linear transformation U on Φ into itself such that

$$U^r\varphi = \varphi \circ f, \qquad \varphi \in \Phi,$$

then U is called the *phantom iterative root* (of order r) of f.

In [I.40] phantom iterative roots are found in a special case. With enough phantom roots one can introduce a continuous phantom iteration group.

We suggest that phantom iterates be called *Targonski iterates.*

Meetings, scientific visits

Professor Targonski liked to take part in mathematical meetings. He always wanted to be among the first ones to arrive and the last ones to leave.

His most important meetings were, of course, the European Conferences on Iteration Theory. But he also did attend almost all of the thirty six annual ISFE, originated and organized by Professor János Aczél (University of Waterloo, Canada). He was present on the International Mathematical Congresses (IMC) in Nice 1970, in Helsinki 1978 and in Zürich 1994, and this August he intended to take part in the IMC in Berlin. He also participated in the ICFEI organized by the Institute of Mathematics of the Pedagogical University of Kraków, in 1991 at Krynica, and in 1995 and 1997 at the "Geofizyk" in Muszyna-Złockie.

He was a member of the American Mathematical Society (for almost fourty years, a Life Member since 1994), Bolyai János Matematikai Társulat, Deutsche Mathematiker-Vereinigung, London Mathematical Society and Société Mathématique Suisse, the Austrian, Canadian, French and Irish mathematical societes and of the European Mathematical Society as well as of the Société Suisse de Physique and of the European Physical Society.

He was invited for different intervals of time to several research institutions and universities in Austria, England, France, Hungary, Italy, Poland, Spain, Switzerland, in Canada and in the U.S.A.

As a result of Professor Targonski's worldwide contacts there is a long list of his friends, working and personal ones. A good part of them was present at the ECIT 98. From among others let us mention here János Aczél. György often said: *from János I learned a lot during the last thirty years.*

In his Curriculum Vitae György Targonski wrote in 1979:

Von 1973 an habe ich eine Zusammenarbeit mit der Forschungsgruppe Dynamische Systeme der Universität Toulouse III (R. Thibault, I. Gumow-

ski, C. Mira und ihre Mitarbeiter) aufgebaut. Die Gruppe in Toulouse interessiert sich hauptsächlich für Anwendungen der Iterationstheorie in Elektrotechnik, Biologie und Physik.

Professor Christian Mira pointed out at ECIT 98 that Győrgy essentially encouraged him to organize the international conference "Point Mappings and Their Applications" (Toulouse, September 10-14, 1973) – a prototype of the ECIT.

In the "Preface" to [I.36] Gy. Targonski wrote on his sabbatical in 1983:

My visits to Los Alamos National Laboratory, to the La Jolla Institute, to Washington University (St. Louis) and to McGill University (Montreal) made me acquainted with many new ideas and problems and involved me in many enjoyable discussions.

The last and longest of my visits was to the University of Graz. The warm welcome by Ludwig Reich and his colleagues and associates, their kind interest in my work and the intelectual atmosphere of the Institute of Mathematics made my stay there most pleasant and useful.

Professor Targonski had several research contracts granted by the: US Air Forces, National Science Foundation, Fonds National Suisse pour la Recherche Scientifique, and European Research Projects. Some of them were to build up contacts and cooperation between mathematicians working on similar fields in different mathematical institutions in Europe (Austria, France, Germany, Hungary, Italy, Poland, Portugal and Spain).

Doctoral students

Professor Győrgy Targonski was very proud of his Ph. D. students. Abe Sklar wrote in a letter sent to the first author in August 1998: *as to his work, I think that, with characteristic self-effacement he would have said that his most lasting achievements were organization of ECIT, and the results produced by his students.*

The doctors promoted by Professor Targonski are:

1. Gisela Zimmermann, *Über die Existenz iterativer Wurzeln von Abbildungen*, 1978.
2. Uhland Burkart, *Zur Charakterisierung diskreter dynamischer Systeme*, 1978.
3. Reinhard Graw, *Über die Orbitstruktur stetiger Abbildungen*, 1978.
4. Sabine Müllenbach, *Contribution à l'étude de l'itération fractionnaire des endomorphismes*, 1983.
5. Jürgen Weitkämper, *Konjugation quadratischer Polynome*, 1988.
6. Reginald Ferber, *Räumliche und zeitliche Regelmässigkeiten zellulärer Automaten*, 1988.

S. Müllenbach was promoted at L'Université Paul Sabatier de Toulouse, and the others had obtained their degrees from the Philipps-Universität Marburg. Each of Targonski's Ph. D. students achieved the degree in a relatively short time. He took them to conferences where they presented their works. Results of the first three are reported in the book [I.34]. The last two doctors continue research work, J. Weitkämper in Oldenburg and R. Ferber in Darmstadt.

European Conferences on Iteration Theory

A favourite among György Targonski's projects was the initiation and subsequent organization of all the European Conferences on Iteration Theory. The first one, not called ECIT yet, was held in Amöneburg, Germany, May 14-18, 1980. The ECIT 98 was the ninth one. Here we only list the preceding ECITs by year and place: 1984, Lochau (Austria); 1987, Caldes de Malavella (Spain); 1989, Batschuns (Austria); 1991, Lisbon (Portugal); 1992, Batschuns (Austria); 1994, Opava (The Czech Republic); 1996 Urbino (Italy).

There were regularly published Proceedings, alltogether 7 volumes, from ECIT 84 onwards, cf. Part III of the Bibliography. Professor Targonski was an editor of the first six volumes, in fact, he was the Managing Editor.

In his last letter, of November 10, 1997, sent to Professors Christian Mira, Ludwig Reich, Norbert Netzer, Luigi Paganoni, Jaroslav Smítal, Claudi Alsina and Bogdan Choczewski (for information), György Targonski indicated the ideas of further meetings, writing:

Our program for the next years is fixed: ECIT 98 in Poland, ECIT 2000 in Austria. For ECIT 02 (in 2002) I have personal hope that our Portuguese friends (who so succesfully organized ECIT 91 in Lisbon) can help again.

Non-mathematical activities

As a Hungarian refugee György Targonski worked for a few years since 1957 in the Hungarian Students and Alumni Organization in Exile.

In 1963 he joined the Hungarian Pax Romana (Katolikus Magyar Értelmiségi Mozgalom) in Exile and started a very active life in the organization. He was its President for seven years (1967-1974). He also organized the local Hungarian Pax Romana while he was working in New York. He took part in the organization of the annual conferences of the Pax Romana movement. He has given several lectures on these conferences, dealing with relations between modern sciences and the Catholic church. He had represented the

Hungarian Pax Romana with two other members of the movement at the audience of Pope Paul VI in 1968 in connection with the general Pax Romana Conference which was held in Rome that year. He belonged to the Board of Editors of the Hungarian Catholic Quarterly "Mérleg" from 1974 to 1986.

György Targonski always followed the life story of people who were victims of some oppressive politics. Through his life he learned to appreciate and fight for the freedom of other people. On several occassions he has given lectures on some subjects concerning political or other problems existing at that time in Hungary under Soviet occupation. He was a supportive member of the German Chapter of Amnesty International and also supported the Campaigns for Human Rights of Professor Israel Halperin (Toronto, Canada).

Polish traces and paths

György Targonski was a very devoted Hungarian. But he was also very proud of his Polish father. When writing to Polish friends he always used the orthography: "Targoński" of his name, with the – existing only in the Polish language – vocal "ń" and even was surprised when getting letters from Poland addressed "Targonski". Once, when visiting Poland, he wanted to find the place of his noble ancestors. His name comes from the village "Targonie" and the family has the "Kościesza" coat-of-arms. Professor Roman Ger reported at ECIT 98 that one year he found Targonie, and sent to György a letter with the stamp of a local authority documenting the existence of the village.

In Poland he had many friends who enjoyed his presence at the conferences held in Poland and his scientific visits to Polish research centres of functional equations and iteration theory in Katowice, Kraków, and Bielsko-Biała.

He was in very good and special contacts with Professor Marek Kuczma (†1991) and his wife Krystyna (†1993). They spent lots of time together. He enjoyed a one year stay of the Kuczmas in Marburg. Kuczma visited Targonski for the first time in New York City in 1968. Both had a special somewhat sarcastic humour. They were very good friends indeed. Targonski's fate was to some extent similar to that of Kuczma, we mean the two strokes each of them suffered and their dedicated fight against the illness.

Close cooperation and friendship linked György Targonski with Marek C. Zdun. Their common stay in Graz in 1986 resulted in the finishing of the booklet [I.40] on which they had started to work in 1978 and a major part [I.33] of which written in 1981, was circulating as a preprint of the Institute des Hautes Études Scientifiques (Bures-sur-Yvette, France). Even after the second brain injuries at the end of 1992 György was thinking of writing a

joint book with Marek, and already had started to work on the structure of chapters.

The first author reported when presenting the paper on the ECIT 98: *I had met György Targonski for the first time in Waterloo (Canada) on the 7th ISFE, 1969. From that time we used to meet each other on most of the further ISFE. I had the pleasure of visiting him in Marburg, once when the Kuczmas were staying there. I always was invited to stay in György's apartment on the 22nd floor in Frankfurt, on my way to Oberwolfach. He and Jolán were so kind to me at the ECIT 94 at Opava in the difficult time in my life. György was an exceptional Man and Friend.*

Mathematician, Hungarian, European, and Teacher

Professor Targonski's academic career lasted more than fourty years in five countries.

He was a very good and beloved teacher. He was willing to to help and support not only the so-called talented students but also those who had difficulties with mathematics. His way of teaching and explaining was very clear and simple. He was very punctual and always kept his time. He often said: *a good mathematician should also be a good teacher.* He believed in this very strongly.

Professor Targonski was a devoted mathematician. Abe Sklar wrote in the letter already quoted:

My own feeling is that the major significance of his work lies in the pioneering effort to establish deep connections between seemingly diverse areas of mathematics. His efforts to link functional equations, iteration theory, and dynamical systems have borne fruit in ECIT, but his efforts in other directions have not really been followed up. How many functional analysts, for example, even know of the existence of the paper "Zur Klassifizierung der linearen Operatoren auf Funktionenalgebren" [cf. [I.21]], called "extremely interesting" by M. Kuczma in Math. Reviews 35, March 1968? Similarly, how many automata theorists know about his lectures [cf. [II.21] and [II.22]] relating automata and functions? To sum up: I feel that György Targonski's major mathematical legacy lies in the areas he has opened up to fruitful and well-deserved investigations.

György Targonski had an excellent command of Hungarian, German, English and French (even if he claimed not to know French properly). He was sorry he had no time and energy to learn Polish. In spite of living so long outside his country he always considered himself as Hungarian. He loved Hungary and was very proud of it. He adhered fervently to the idea of a United Europe. The blue European flag he carried in his luggage from ECIT to ECIT is a visible sign of this adherence.

György Targonski did not take himself too seriously. But there were a few things in his life he did take very seriously: to be a mathematician, to be Hungarian and European. And, in spite of his bad health and other difficulties in his life, he did not lose his faith in God. Till the last minute of his life.

Acknowledgements. We want to conclude the paper with thanks to Győrgy's friends: Adolf Schleiermacher, Abe Sklar and Marek Cezary Zdun for supporting us with opinions and materials which have been used in the preparation of the paper. Thanks are extended to Mr. W. Wilk for his assistance, in particular, for the professional scanning of pictures and to Miss E. Dudek, Miss J. Wiercioch and Mrs. A. Grabiec for their help in computer setting of the text of the article.

The mathematical works of Győrgy Targonski

I. Research papers and reports, books, lecture notes

1. Az iterációszámításról [On the Iteration Calculus]. Budapest University, 1951. Unpublished.

2. Két formula a géodéziai alapvonalhosszuságok redukciójához [Two Formulae for the Reduction of Basic Distances in Geodesy]. Hungarian Institute of Cartography, Budapest, 1952. Unpublished.

3. *Darstellungen von Funktionen durch Kettenreihen.* Publ. Math. Debrecen **2** (1952), 286–290.

4. (with Z. Fekete) *Kombinatorika* [Combinatorics]. Tankönyvkiadó, Budapest 1952.

5. *An always convergent iteration process.* Acta Math. Acad. Sci. Hungar. 4 (1953), 119–126.

6. (with Z. Bognár) *Über die Bestimmung konjugierter harmonischer Funktionen.* Publ. Math. Debrecen **3** (1954), 215–216.

7. Végtelen sorozatok az iterációelméletben [Infinite Series in the Theory of Iteration]. Thesis for the Candidature of Sciences. Budapest University, 1955. Unpublished.

8. *Interpolation durch Reihen iterierter Funktionen.* Commentationes Physico-Mathematicae **20**, 9. Helsinki 1957, 3–12.

9. *Approximation of velocity-dependent potentials by separable kernels.* Il Nuovo Cimento, Ser. X, 4 (1959), 1093–1101.

10. On Substitution Operators. I. A Summation Method. Technical Report No. AF 61 (052) – 320. Air Force Office of Scientific Research, Queen Mary College, University of London 1960. Unpublished.

11. (with A. Martin) *On the uniqueness of a potential fitting a scattering amplitude at a given energy.* Il Nuovo Cimento, Ser. X, **20** (1961), 1182–1190.

12. (with B. Misra and D. Speiser) *Integraldarstellung von Operatoren in der Streutheorie.* Helvetica Phys. Acta **35** (1962), 329–330.

13. Contributions to the Theory of Scattering, Cambridge University, 1963, Ph.D. thesis.

14. (with B. Misra and D. Speiser) *Integral operators in the theory of scattering.* Helvetica Phys. Acta **36** (1963), 963–980.

15. *On a theory of linear functional equations.* Proceedings of the Fourteenth Scandinavian Mathematical Congress, Copenhagen 1964.

16. A Bibliography on Functional Equations with Abstracts. U.S. Air Force Office of Scientific Research Report, New York 1964. Unpublished.

17. *Schröder's equation and a generalization of Chebyshev polynomials*, Trans. New York Acad. Sci. **27** (1965), 600–605.

18. *Seminar on Functional Operators and Equations.* Part I and II. Eidgenössische Technische Hochschule, Zürich, 1966.

19. *On Carleman integral operators.* Proc. Amer. Math. Soc. **18** (1967), 450–456.

20. *Convergence theorems derived from the theory of Carleman integral operators.* Compositio Math. **18** (1967), 148–154.

21. *Zur Klassifizierung der linearen Operatoren auf Funktionenalgebren.* Math. Zeitschr. **97** (1967), 238–250.

22. *Seminar on Functional Operators and Equations.* Springer Lecture Notes in Mathematics, No. 33. Springer-Verlag, Berlin, Heidelberg, New York, 1967.

23. A short survey of some recent results on integral operators. Nat. Science Found. Grant No. GP. 8757, 1968. Unpublished.

24. (with M. Schreiber) *Carleman and semi-Carleman operators.* Proc. Amer. Math. Soc. **24** (1970), 293–299.

25. *Linear endomorphisms of function algebras and related functional equations.* Indiana Univ. Math. J. **20** (1970), 579–589.

26. (with M. Kuczma) *On a Pre–Schröder equation.* Bull. Acad. Polon. Sci., Sér. sci. math. astr. phys. **18** (1970), 721–724.

27. *Über die lineare Funktionalgleichung erster Ordnung.* Jber. Deutsch. Math.–Verein. **73** (1971), 101–110.

28. *Charakterisierung von Operatoren durch Funktionalgleichungen.* Demonstratio Math. **6** (1973), 861–870.

29. *An iteration theoretical approach to the concept of time.* Proceedings of the International Conference on Point Mapping and its Applications, Toulouse, September 10-14, 1973. CNRS 1976, 245–257.

30. *Orbit properties of functions and "Pre–Abel" equations.* Ann. Polon. Math. **33** (1977), 49–55.

31. *Einige Anwendungen des Orbitbegriffs von Kuratowski.* Ber. der math.--stat. Sekt. Forschungszentrum Graz, Ber. Nr. **83**, 1978.

32. *On orbit theory and some of its applications.* Zeszyty Nauk. Akad. Górn.-Hutniczej im. St. Staszica Nr. 764, Mat.-Fiz.-Chem. **43** (1980), 7–14.

33. (with M. C. Zdun) *Substitution Operators on L^p-spaces and Their Semigroups.* Institut des Hautes Etudes Scientifiques. Preprint IHES/M/81/33. Bures-sur-Yvette, 1981.

34. *Topics in Iteration Theory.* Vandenhoeck & Ruprecht, Göttingen – Zürich, 1981.

35. *Unsolved problems in iteration theory.* In: Théorie de l'Iteration et ses Applications, Toulouse, 17–22 Mai, 1982. Editions du centre national se la recherche scientifique, 1982, 23–32.

36. *New Directions and Open Problems in Iteration Theory.* Ber. der math.-stat. Sekt. Forschungszentrum, Graz, 1984. Ber. Nr. **229**, 1984.

37. *Iterationstheorie zwischen Funktionalgleichungen und Dynamik.* Jahrbuch Überblicke Mathematik 1985, 9–28.

38. *Embedding of discrete dynamical systems.* In: Demongeot, J.; Golès, N.; Thuente, M. (editors), Dynamical Systems and Cellular Automata. Academic Press, London, San Diego, New York, Boston, Sydney, Tokyo, Toronto, 1985, 65–72.

39. (with M. C. Zdun) *Generators and co-generators of substitution semigroups.* Annal. Math. Siles. **1(13)** (1985), 169–174.

40. *Phantom iterates of continuous functions.* In [III.1] (ECIT 84), 196–202.

41. (with M. C. Zdun) *Substitution Operators on L^p–spaces and Their Semigroups.* Ber. der math.-stat. Sekt. Forsch. Ges. Joanneum Graz. Ber. Nr. **283**, 1987.

42. *Iteration theory and its functional equations.* Rendiconti del Seminario Matematico e Fisico di Milano **58** (1988), 207–220.

43. *Iteration Theory and Functional Analysis.* In [III.2] (ECIT 87), 74–93.

44. *On Composition Operators.* Fachbereich Mathematik, Universität Marburg. Bericht No. 1 (1989). Preprint.

45. (with C. Alsina and J. L. García-Roig) *A remark on Pythagoras Theorem and functional conjugacy.* In [III.3] (ECIT 89), 6–8.

46. (with R. Ferber and J. Weitkämper) *Fractional–time states of cellular automata.* In [III.3] (ECIT 89), 86–106.

47. *On composition operators.* Zesz. Nauk. Pol. Śl. nr. 1070. Seria: Matematyka–Fizyka **64** (1991), 229–234.

48. *On a class of phantom fractional iterates.* In [III.4] (ECIT 91), 295–304.

49. *Functional equations connected with phantom iterates.* Opuscula Math. 14 (1994), 179–182.

50. *Progress in iteration theory since 1981*, Aequationes Math. **50** (1995), 50–72.

II. Talks, Problems, Remarks

1. *Lineare Operatoren auf Funktionenalgebren* [Abstract of talk]. In: Die vierte Tagung über Funktionalgleichungen, Oberwolfach, 10.-16. Juli 1966 (comp. by M. Kuczma). Aequationes Math. 1 (1968), 136–137.

2. *2. Problem. Verallgemeinerte Tschebyscheffsche Polynome.* Ibidem, 146.

3. *On a solution method for Schröder's equation* [Abstract of talk]. In: Fifth Annual Meeting on Functional Equations, Waterloo, Ont., April 24–29, 1967 (comp. by M. Kuczma). Aequationes Math. **1** (1968), 283–284.

4. *8. Problem* [Generalized Chebyshev polynomials]. Ibidem, 302.

5. *9. Remark* [On the "commuting fixed point" conjecture]. Ibidem, 302.

6. *Endomorphisms of function algebras and Schröder's equation* [Abstract of talk]. In: Die sechste Tagung über Funktionalgleichungen, Oberwolfach, 17–22 Juni, 1968 (comp. by I. Fenyö). Aequationes Math. **2** (1969), 361.

7. *14. Problem. Characterization of harmonic functions.* Ibidem, 372.

8. *16. Remark on the paper read by J. Aczél.* Ibidem, 373–374.

9. *21. Remark. On the set of solutions of the translation equation.* Ibidem, 375–376.

10. *A short survey of some recent results on integral operators.* Talk given at the Symposium on Advanced Topics in the Mathematical Foundations of Scattering Theory, Geneva, Switzerland, July 1968. Cf. [I.23].

11. *Linear endomorphisms of function algebras* [Abstract of talk]. In: Seventh International Symposium on Functional Equations, September 1-13, 1969 (comp. by F. Radó). Aequationes Math. 4 (1970), 219–220.

12. *13. Remark to Professor E. Hille's lecture.* Ibidem, 245.

13. *23. Problem* **(P63)** [The Pre-Schröder System]. Ibidem, 251.

14. **P63R1** [Problem **P63** reformulated]. Aequationes Math. 4 (1970), 395.

15. *Iteration theory and trajectories in state spaces* [Abstract of talk]. In: Achte internationale Tagung über Funktionalgleichungen in Oberwolfach von 2–8 August 1970 (comp. by W. Leissner). Aequationes Math. **6** (1971), 104.

16. *10. Remark* [On results on the Pre-Schröder System]. Ibidem, 108.

17. *On Pre-Schröder Equations* [Abstract of talk]. In: Ninth International Symposium on Functional Equations, September 3–11, 1971 (comp. by F. Neuman). Aequationes Math. **8** (1972), 157.

18. *10. Remark (in connection with Prof. Schatten's talk).* Ibidem, 173.

19. *Pre-Abel Equations* [Abstract of talk]. In: Elfte internationale Tagung über Funktionalgleichungen in Oberwolfach vom 14. bis 20. Dezember 1973 (comp. by W. Eichhorn). Aequationes Math. 11 (1974), 304–305.

20. *16. Remark to the Talk of R. Beauvais.* Ibidem, 313.

21. *Problems related to the talk of A. Smajdor* (**P127**). Ibidem, 314.

22. *Continuous functions as countable automata* [Abstract of talk]. In: Dreizehnte internationale Tagung über Funktionalgleichungen in Oberwolfach vom 6.7 bis 12.7.1975 (comp. by M. Kuczma). Aequationes Math. 14 (1976), 210–211.

23. *Representations of functions as automata* [Abstract of talk]. In: Fourteenth international symposium on functional equations, May 21–28, 1976 (comp. by C. Sempi). Aequationes Math. 15 (1977), 269.

24. *Iteration theory and topological dynamics* [Abstract of talk]. In: Fifteenth international symposium on functional equations, Oberwolfach, May 22–28, 1977 (comp. by F. Neuman). Aequationes Math. 17 (1978), 371–372.

25. *Problem* (**P172**) [Nontrivial solutions of Schröder's equation in an algebra of polynomials in two variables]. Ibidem, 373.

26. (with M. C. Zdun) *Characterization of endomorphisms and semigroups of substitution operators in L^p-spaces* [Abstract of talk]. In: Sechzechnte Internationale Symposium über Funktionalgleichungen (comp. by J. Schwaiger). Aequationes Math. 19 (1979), 279–280.

27. *5. Remark* [On an approach to iteration problems of a self-mapping of a countable set]. Ibidem, 285.

28. *6. Problem* (**P180**) [Roots of a matrix having 0 entries except for one 1 in each row]. Ibidem, 285.

29. *7. Remark* [On non-embeddability of homeomorphisms of n-dimensional space to itself into a continuous one-parameter group of homeomorphisms (result joint with R. Graw)]. Ibidem, 285–286.

30. *17. Bemerkung* [Correction to the paper [I.27]]. Ibidem, 293.

31. *25. Remark on work of E. Gier on automata representing real functions.* Ibidem, 297.

32. *21. Remark and problem* (**P201**) [1) Composing functions with given orbit structures. 2) Dependence of orbit structure of composed functions on the order of composition]. In: Siebzehnte internationale Tagung über Funktionalgleichungen in Oberwolfach vom 17.6. bis 23.6.1979 (comp. by H.-H. Kairies). Aequationes Math. 20 (1980), 311–312.

33. *Some current directions of research in iteration theory* [Introductory lecture]. Iteration Theory and Applications, Amöneburg, 14–18 May 1980.

34. (with M. C. Zdun) *Generators and co-generators of substitution semigroups in Banach spaces* [Abstract of talk]. Proceedings of the Eighteenth International Symposium on Functional Equations (comp. by L. Redlin), Centre for Information Theory, Faculty of Mathematics, University of Waterloo, Waterloo, Ont., Canada, 1980, 27–28.

35. *Iteration Theory* [Special Session organized by Gy. Targonski]. Ibidem, 46.

36. *Topological entropy* [Abstract of talk]. Proceedings of the Nineteenth International Symposium on Functional Equations, Nantes and La Turballe, May 3–13, 1981 (comp. by L. Redlin), Centre for Information Theory, University of Waterloo, Waterloo, Ont. Canada, 1981, 50.

37. *Iteration Theory* [Special Session organized by Gy. Targonski]. Ibidem, 91–92.

38. *Directed graphs and the structure of chaos: results of U. Burkart* [Abstract of talk]. In: The Twentieth International Symposium on Functional Equations, August 1 – August 7, 1982, Oberwolfach, Germany (comp. by B. R. Ebanks). Aequationes Math. **24** (1982), 285–286.

39. *Linearization of iteration problems* [Abstract of talk]. In: The Twenty--first International Symposium on Functional Equations, August 6 - August 13, 1983, Konolfingen, Switzerland (comp. by F. Zorzitto). Aequationes Math. **26** (1983), 251–252.

40. *Embedding of discrete dynamical systems.* Talk given at the "Journées de la Sociéteé Mathématique de France" on "Dynamical Behaviour of Cellular Automata: Theory and Applications", September 13–17, 1983, Luminy, France. Cf. [I.38].

41. *A functional equation for phantom iterative roots* [Abstract of talk]. In: The Twenty-second International Symposium on Functional Equations, December 16 – December 22, 1984, Oberwolfach, Germany (comp. by B. Ebanks). Aequationes Math. **29** (1985), 82–83.

42. *34. Problem* (**P274**) [The Jabotinsky and the Translation Equations]. Ibidem, 106.

43. *Iteration Theory* [Special Session organized by L. Reich and Gy. Targonski]. Proceedings of the Twenty-third International Symposium on Functional Equations, Gargnano, Italy, June 2 – June 11, 1985 (comp. by T. M. K. Davison). Centre for Information Theory, Faculty of Mathematics, University of Waterloo, Waterloo, Ont., Canada, 1985, 80–81.

44. *Phantom states of dynamical systems* [Abstract of talk]. In: The Twenty--fourth International Symposium on Functional Equations, August 12 – August 20, 1986, South Hadley, MA (comp. by M. Sablik). Aequationes Math. **32** (1987), 122.

45. *Iteration Theory* [Special Session organized by L. Reich, A. Sklar and Gy. Targonski]. Ibidem, 147–148.

46. *Phantom roots of real square matrices: do they exist?* [Abstract of talk]. In: The Twenty-fifth International Symposium on Functional Equations, August 16–22, 1987, Hamburg-Rissen, Germany (comp. by R. Ger). Aequationes Math. **37** (1988), 107.

47. *Phantom dynamics* [Abstract of talk]. In: The Twenty-sixth International Symposium on Functional Equations, April 24 – May 3, 1988, Sant Feliu de Guixols, Catalonia, Spain (comp. by B. Ebanks). Aequationes Math. **37** (1989), 91.

48. *Iteration Theory* [Special Session organized by Z. Moszner, L. Reich and Gy. Targonski]. Ibidem, 123–124.

49. *Open questions about KW-orbits and iterative roots* [Abstract of talk]. In: The Twenty-eighth International Symposium on Functional Equations, August 23 – September 1, 1990, Graz-Mariatrost, Austria (comp. by J. Schwaiger). Aequationes Math. 41 (1991), 277–278.

50. *14. Problems* [Self-mappings of the plane which are conjugate to a complex polynomial. 2) The Kuratowski-Whyburn orbit structure "of polynomial type"]. Ibidem, 289.

51. *20. Remark* [On "L-soft functional equations"]. Ibidem, 292.

52. *The Translation Equation and Iteration* [Special Session organized by Z. Moszner and Gy. Targonski]. Ibidem, 310.

53. *Iteration Theory in 1992.* [Survey lecture on the 30th International Symposium on Functional Equations, Oberwolfach, Sept. 20–26, 1992]. Cf. [I.50].

54. *2. Problem concerning the third Jabotinsky equation.* In: The Thirtieth International Symposium on Functional Equations, September 20–26, 1992, Oberwolfach, Germany (comp. by K. Nikodem). Aequationes Math. **46** (1993), 283.

55. *Phantom Iterates and Liedl's Pilgerschritt Transformation.* [III.5] (ECIT 92), 284.

56. *Problems of iteration theory - still unsolved* [Abstract of talk] In: The Thirty-third International Symposium on Functional Equations, May 21 – May 27, 1995, Caldes de Malavella, Catalonia, Spain (comp. by M. Sablik). Aequationes Math. **51** (1996), 160.

57. *Introducing the "Pre-Eigenvalue Equations"* [Abstract of talk]. In: The Thirty-fourth International Symposium on Functional Equations, June 10 to 19, 1996, Wisła-Jawornik, Poland (comp. by R. Badora). Aequationes Math. **53** (1997), 182.

58. *Some unsolved problems in Iteration Theory.* [III.6] (ECIT 94), 251.

59. *Functional equations connected with the second automaton equation* [Abstract of talk]. In: The Thirty-fifth International Symposium on Functional Equations, September 7–14, 1997, Graz-Mariatrost, Austria (comp. by J. Sikorska). Aequationes Math. **55** (1998), 295.

III. Proceedings of the European Conferences on Iteration Theory

1. (ECIT 84) Liedl, R., Reich, L., Targonski, Gy. (editors), *Iteration Theory and Its Functional Equations. Proceedings of the International Conference, Schloß Hofen (Lochau, Austria), September 28 – October 1, 1984.* Lecture Notes in Mathematics 1163. Springer-Verlag, Berlin, Heidelberg, New York, Tokyo 1985.

2. (ECIT 87) Alsina, C., Llibre, J., Mira, C., Simó, C., Targonski, Gy. (editors). *European Conference on Iteration Theory (ECIT 87), Caldes de Malavella, Spain, September 20–26, 1987.* World Scientific, Singapore, New Jersey, London, Hong Kong, 1989.

3. (ECIT 89) Mira, C., Netzer, N., Simó, C., Targonski, Gy. (editors), *European Conference on Iteration Theory (ECIT 89), Batschuns, Austria, 10–16 September 1989.* World Scientific, Singapore, New Jersey, London, Hong Kong, 1991.

4. (ECIT 91) Lampreia, J. P., Llibre, J., Mira, C., Sousa Ramos, J. and Targonski, Gy. (editors), *European Conference on Iteration Theory (ECIT 91) Lisbon, Portugal, 15–21 September, 1991.* World Scientific, Singapore, New Jersey, London, Hong Kong, 1992.

5. (ECIT 92) Förg-Rob, W., Gronau, D., Mira, C., Netzer, N., Targonski, Gy. (editors) *Iteration Theory. Proceedings of the European Conference, Batschuns, Austria, 13–19 September, 1992.* World Scientific, Singapore, New Jersey, London, Hong Kong, 1996.

6. (ECIT 94) Reich, L., Smítal, J. and Targonski, Gy. (editors), *Iteration Theory. Proceedings of the European Conference on Iteration Theory, Opava, The Czech Republic, Aug. 28 - Sept. 3, 1994.* Grazer Math. Ber. Nr. 334 (1997).

7. (ECIT 96) Forti, G. L., Gardini, L., Gronau, D., Paganoni, L. (editors) *Iteration Theory. Proceedings of the European Conference on Iteration Theory, Urbino, Italy, Sept. 8–14, 1996.* Grazer Math. Ber. (1999), to appear.

Institute of Mathematics
Pedagogical University
Podchorążych 2
30-084 Kraków
Poland

Rablstrasse 18
D-81 669 München
Germany