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ON A DECOMPOSITION OF THE WITT GROUP INTO DIRECT SUM OF CYCLIC GROUPS

Abstract. In the paper it is shown that the Witt group of the rational function field in countably many variables over a real-closed field can be decomposed into direct sum of cyclic groups. This is an example that the sufficient conditions given in [3] are not necessary.

The problem whether the Witt group of a field can be decomposed into direct sum of cyclic groups or not, was considered in [3]. There Szymiczek also proved that this problem can be reduced to the torsion subgroup and in the case of a countable or bounded torsion subgroup the decomposition into direct sum of cyclic groups is possible. In particular it occurs in the case of the Witt group of a non-real field. There is still no example of indecomposable Witt group and the decomposability is not proved. The example given below does not change the situation but it is the example of a decomposable Witt group with uncountable and unbounded torsion subgroup.

THEOREM. *Let $F = \mathbf{R}(x_1, x_2, \dots)$ be the field of rational functions in countably many variables with coefficients in a real-closed field \mathbf{R} . Then the group $W_t F$ can be decomposed into direct sum of cyclic groups.*

REMARK 1. *If $|\mathbf{R}| = m$ then $|W_t F| = m$.*

It suffices to consider the set $\{ \langle 1, -(x^2 + c) \rangle : c \in \mathbf{R} \text{ and } c > 0 \}$ by proving Remark 1.

Proof of the Theorem. We use the Kulikov criterion (cf. [1]) for the subgroups $A_n = W_t \mathbf{R}(x_1, x_2, \dots, x_n)$. It is obvious that $\bigcup \{A_n : n \in \mathbf{N}\} = W_t F$, so we must show that the heights of non-zero elements of A_n in A_n have a common bound. It remains to show that the height does not grow under the extension $A_n \subset A_{n+1}$. Indeed, let Φ be a non-zero form in A_n and τ a form defined over $\mathbf{R}(x_1, x_2, \dots, x_n, x_{n+1})$ such that $\Phi = 2^r \cdot \tau$. Using the homomorphism $\lambda: W_t \mathbf{R}(x_1, x_2, \dots, x_{n+1}) \rightarrow W_t \mathbf{R}(x_1, x_2, \dots, x_n)$ defined by the suitable place we get

$$\Phi = \lambda(\Phi) = 2^r \cdot \lambda(\tau).$$

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By the order argument we get

$$2^r < 2^n \quad \text{and} \quad r < n.$$

Thus the theorem follows.

REMARK 2. Using the theorem of Pfister (cf. [2, Ch. 4, Theorem 2.1]) one can replace \mathbf{R} in the above Theorem by an arbitrary finite transcendence degree extension of it.

REFERENCES

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