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A BASIC SET OF CANCELLATION VIOLATING SEQUENCES FOR FINITE TWO-DIMENSIONAL NON-ADDITIVE MEASUREMENT

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Dedicated to Professor Kazimierz Nikodem on the occasion of his 70th birthday

Abstract. Cancellation conditions play a central role in the representation theory of measurement for a weak order on a finite two-dimensional Cartesian product set X. A weak order has an additive representation if and only if it violates no cancellation conditions. Given X, a longstanding open problem is to determine the simplest set of cancellation conditions that is violated by every linear order that is not additively representable. Here, we report that the simplest set of cancellation conditions on a 5 by 5 product X is obtained.

1. Background, Introduction

Let $\succ$ be a linear order on a product set $X = X_1 \times X_2$ with finite X_1 and X_2. The linear order has an additive representation if there exist maps $u_1\colon X_1 \to \mathbb{R}$, $u_2\colon X_2 \to \mathbb{R}$ such that

$$(x_1, x_2) \succ (y_1, y_2) \quad \text{iff} \quad u_1(x_1) + u_2(x_2) > u_1(y_1) + u_2(y_2).$$

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It is well known that $\succ$ has an additive representation if and only if no finite sequence $x^1 \succ y^1$, $x^2 \succ y^2$, …, $x^J \succ y^J$ has the cancellation property that x_i^1, x_i^2, …, x_i^J is a permutation of y_i^1, y_i^2, …, y_i^J for $i = 1, 2$.

With this opening introduction (adapted for the two-dimensional X), Fishburn [1] points out (in the abstract) a longstanding open problem – to determine the simplest subset of cancellation conditions that is violated by every linear order that is not additively representable.

Let S denote a given class of non-additive (a short form for not additively representable) weak orders on X. We may pose the above open problem in a broader setting – to determine a simplest subset of cancellation conditions that is violated by every weak order belonging to S. We shall denote such a subset by $\mathcal{C}(S)$ for speedy reference – keeping in mind that it may have multiple meanings.

There is some leeway in the interpretation of the open problem. We need to clarify what "simplest" means. Reading further into Fishburn [1], it is apparent that if a weak order violates two cancellation conditions, the one with shorter width is regarded as simpler. For the case of where S consists of a single weak order, and that weak order violates two cancellation conditions having the same minimum width, $\mathcal{C}(S)$ has two answers and neither is *the simplest*.

Fishburn, in [2], asks questions whose answers need some good understanding of $\mathcal{C}(S)$ where S is the class of all non-additive weak orders on X with small sizes $m := |X_1|$ and $n := |X_2|$. Both linear and non-linear orders are involved. In particular he gives answers for the cases of $(m, n) = (3, 3), (3, 4), (4, 4)$, and put forward, in the discussion section, the questions for the cases of $(m, n) = (3, 5), (4, 5), (5, 5)$.

In Ng [4] and [5] his questions for $(m, n) = (3, 5), (4, 5)$ get answered. In that process, $\mathcal{C}(S)$ is examined for the class S of all non-additive weak orders on X with sizes (m, n) below $(5, 5)$.

The current article focuses on X with size $(m, n) = (5, 5)$. We report that $\mathcal{C}(S)$ is found for the class S of all non-additive linear orders on X.

The work is very much a continuation of [5]. The setup, the use of terms, and the approach are closely followed. Theorems and definitions cited are mostly from that paper. In particular, weak orders on X are replaced by 5 by 5 arrays with numeric entries. Cancellation sequences (x^1, y^1), (x^2, y^2), …, (x^J, y^J) are presented using figures that show the comparing pairs. Definition 8 clarifies what it means when we say a particular comparing pair is basic for a non-additive order, and what it means when we say a figure (or the cancellation sequence it presents) is basic for the order. Loosely speaking, a figure is basic for a non-additive order (or array) means that the figure is the simplest amongst those that the order violates, and should be included in $\mathcal{C}(S)$ for any S which contains that order. Here, let us add one more terminology to Definition 8.

Definition 8 (extended). A figure is basic for a class S of non-additive weak orders on X if it is basic for at least one weak order belonging to S.

In [5], Theorem 11 is used most often to uncover some basic comparing pairs for any non-additive array. It is Theorem 16, a redacted version of a theorem of Fishburn, that is used most often to confirm that a given figure is basic for a non-additive linear array. In the proof of Proposition 9, there is a demonstration on how Theorem 16 is applied to confirm that a figure is basic for a 4 by 4 (non-additive, understood) linear array. Example 12 illustrates how Theorem 11 is applied to confirm that a figure is basic for a non-linear array. Remark 13 is an example demonstrating how Theorem 11 is used to uncover two different figures which are equally simple for a given 6 by 3 array – both have lowest possible width 6.

2. $\mathcal{C}(S_1)$ for the class S_1 of all non-additive 5 by 5 critical-to-inspect linear arrays

An array is critical-to-inspect if it is non-additive and all its proper subarrays are additive ([5, Definition 6]). Let S_1 be the class of all non-additive 5 by 5 critical-to-inspect linear arrays.

Theorem 2.1. *The set $\mathcal{C}(S_1)$ consists of* (410) *cancellation violating sequences which are represented by figures. The figures are organized into clusters under duality and transposition invariance. There are a total of* (105) *clusters. All* (410) *figures are basic for S_1.*

The full set $\mathcal{C}(S_1)$ is reported in a repository [6, Dataset].

An overview. Here we display the first and the last cluster. Each cluster is accompanied by an array from S_1 which is used to support the claim – that the figures in the cluster are basic for S_1. Notice that the accompanying arrays are not members of the set $\mathcal{C}(S_1)$ and they play only a supportive role.

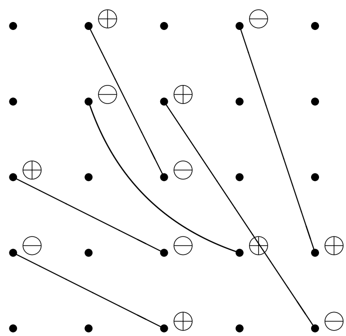


Fig1A

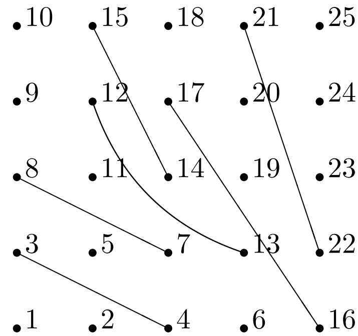


Fig1A (Linear11[1], basic)

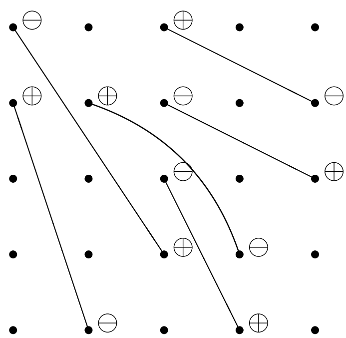


Fig1B (dual of 1A)

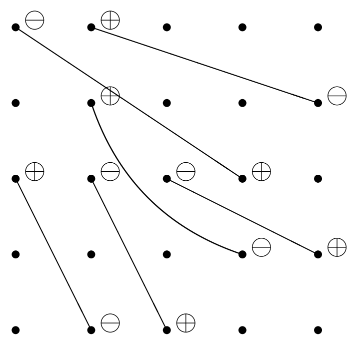


Fig1At (transpose of 1A)

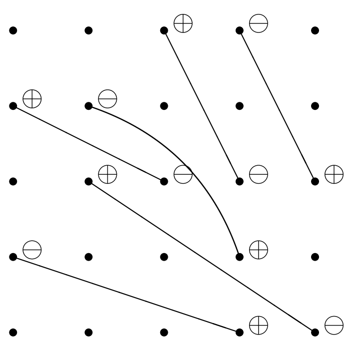


Fig1Bt (transpose of 1B)

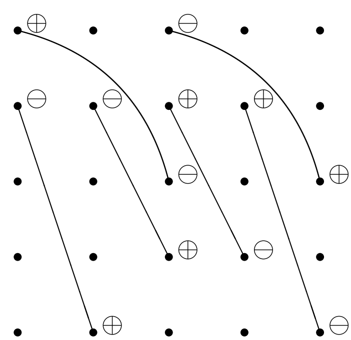


Fig105A

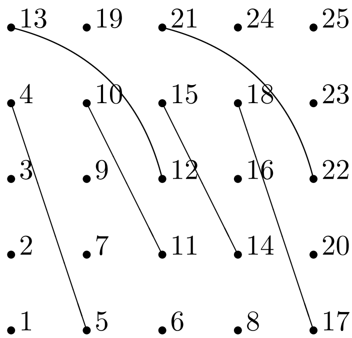


Fig105A (Linear2304[13], basic)

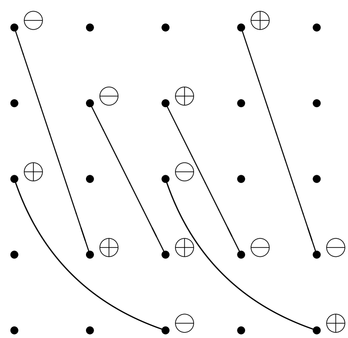


Fig105B

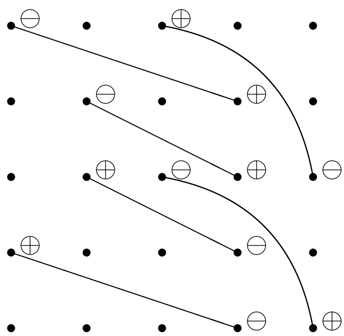


Fig105At

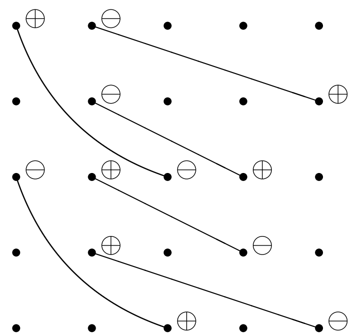


Fig105Bt

The array accompanying Fig1A carries in its caption two pieces of information. Linear11[1] indicates where it is coming from. "Linear11criticaltoinspect.txt" is the actual full filename of the source data file ([6, Dataset]) and [1] indicates that it is the first array on that file. The qualifier basic is to assert that Fig1A is basic for the array. It signals that Fig1A is necessarily a member of $\mathcal{C}(S)$ for any S which contains the array. The super-imposition of the array over the figure allows for easy visual confirmation that the array violates the underlying cancellation condition. The assertion that Fig1A is basic for the array is actually confirmed by an application of Theorem 16 (behind the scene).

The dual of an array is minus 1 times the array, often converted to its canonical form. The dual of Linear11[1] is

$$\begin{bmatrix} 10 & 20 & 22 & 24 & 25 \\ 4 & 13 & 19 & 21 & 23 \\ 3 & 7 & 12 & 15 & 18 \\ 2 & 6 & 9 & 14 & 17 \\ 1 & 5 & 8 & 11 & 16 \end{bmatrix}.$$

The transpose of Linear11[1] is

$$\begin{bmatrix} 16 & 22 & 23 & 24 & 25 \\ 6 & 13 & 19 & 20 & 21 \\ 4 & 7 & 14 & 17 & 18 \\ 2 & 5 & 11 & 12 & 15 \\ 1 & 3 & 8 & 9 & 10 \end{bmatrix}.$$

By the way the figures are labelled, it is understood that Fig1B is basic for the dual of Linear11[1], Fig1At is basic for the transpose of Linear11[1], and Fig1Bt is basic for the transpose of the dual of Linear11[1].

As we learn from the first cluster, that the figures for the transpose and dual can be drawn using symmetries (a reflection along a diagonal line gives the transpose, and a reflection about the center point gives the dual). In our production of the list, once we obtain a figure which is basic for an array from S_1, immediately we draw up the rest and form a cluster using the said symmetries.

The fact that the list is complete follows by a confirmation that every array in S_1 is indeed detected by at least one figure on the list. This is achieved by applying the figure detection worksheet which holds all (410) figures to all arrays in S_1.

2.1. Steps taken to create the data and notes on record keeping

The steps taken to get the above figures are the same as outlined in ([5, §5.2]). Of course, new Maple worksheets are written to suit a 5 by 5 product. There are adjustments to the level of record keeping. We no longer save the generated linear canonical arrays to file after the extraction of the critical-to-inspect ones, partly because there are too many, and partly because it is not time consuming to reproduce. “5by5first10.txt” contains (6484) partially filled linear canonical arrays. Partial in the sense that the generation takes a pause after the first 10 digits are filled. They serve as (6484) leads. Records of the extracted critical-to-inspect arrays are saved under filenames ranging from “Linear1criticaltoinspect.txt” to “Linear6484criticaltoinspect.txt”.

The arrays shown for Fig1A and Fig105A can be read by executing the Maple command lines

```
read ‘‘Linear11criticaltoinspect.txt’’: criticaltoinspect[1];
```

```
read ‘‘Linear2304criticaltoinspect.txt’’: criticaltoinspect[13];
```

Each linear canonical array must carry the digit 2 in the first column or in the first row, adjacent to the digit 1 at the lower left corner. So, either it carries the digit 2 in the first column, or that its transpose carries the digit 2 in the first column. The list of figures we seek will be invariant under transpose. So we may opt to concentrate only on those arrays carrying the digit 2 in its first column. This cuts down on the pursue of the (6484) leads by half. Starting from lead 1200 this work reduction is implemented. The late action is why “Linear11criticaltoinspect.txt” has arrays with a digit 2 in its first row.

Another work reduction is achieved by visually inspecting the leads further before we resume generation. For example, lead 1793, which is obtained by executing the Maple command line

```
read ‘‘5by5First10.txt’’: First10Weakorders[1793];
```

is the array

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 9 & 10 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 & 0 \\ 2 & 5 & 8 & 0 & 0 \\ 1 & 3 & 4 & 0 & 0 \end{bmatrix}.$$

Based on the positions of the digits $2, 3, 4, 6, 7, 8$ we see a violation of the hexagonal condition ([5, Appendix A, Fig. 1]). So, when fully filled, the canonical array has a 3 by 3 sub-array which is non-additive, hence is not critical-to-inspect. We know instantly that the “Linear1793criticaltoinspect.txt” file will carry no array and there is no need to resume generation. Empty files are often removed.

2.2. Observations

Every figure in $\mathcal{C}(S_1)$ is unitary, connected, and thus irreducible. All compared pairs in the supportive arrays are adjacent (consecutively valued). These features provide good environment for the application of Theorem 16. However, we do encounter linear critical-to-inspect arrays which violates more than one simplest cancellation sequence. An example is

$$\begin{bmatrix} 15 & 21 & 22 & 24 & 25 \\ 11 & 16 & 17 & 20 & 23 \\ 8 & 12 & 14 & 18 & 19 \\ 2 & 4 & 6 & 9 & 13 \\ 1 & 3 & 5 & 7 & 10 \end{bmatrix}$$

(source file "Linear6231criticaltoinspect.txt", item [5]). It is detected by Fig28A and Fig79At (not shown here). Both figures have the same width. No figure is basic for this array.

3. $\mathcal{C}(S_2)$ where S_2 is the class of all non-additive 5 by 5 linear arrays

Prelude. Every non-additive linear array, if not already critical-to-inspect, contains some proper non-additive linear sub-array which is. So it violates some basic cancellation conditions known for a smaller size critical-to-inspect array. These cancellation conditions are mounted on the row and column positions taken up by the critical-to-inspect sub-array.

Hence, starting with the above family of (105) clusters of cancellation conditions, and adding to it those basic conditions known for classes of smaller arrays which can be mounted on some sub-products of the current 5 by 5 X, we arrive at a collection which is adequately large – that it is violated by every non-additive linear order on X.

THEOREM 3.1. *$\mathcal{C}(S_2)$ is the disjoint union of two parts. Part 1 is $\mathcal{C}(S_1)$ described in Theorem 2.1, in (105) clusters. Part 2 consists of all the basic cancellation conditions known for smaller size arrays, mounted on (or embedded in) all possible sub-products of the current 5 by 5 product structure X. All conditions are basic for S_2.*

The prelude shows that the stated family is adequately large. To prove the theorem, we need to verify the claim – that all conditions are basic for S_2.

Those in Part 1 are basic for S_1 and hence basic for $S_2 \supset S_1$. We shall proceed to show that every condition in Part 2 is basic for S_2.

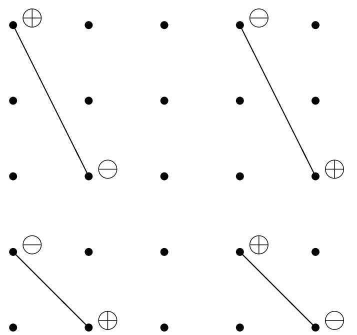
Since the finding for any figure also goes over to its dual and transpose, for each cluster of conditions, we need only pick one to examine. For instance, for the four 4 by 4 Figures 6A to 6D (see §5.1), it is sufficient to check just 6A.

As pointed out earlier, the theorem used most often to confirm that a cancellation condition is basic for a linear array is Theorem 16. The overall theme of that theorem has three components A, B and C. Here A is a cancellation condition, B and C are arrays. The theorem asserts that, when the three components satisfy some specific properties and inter relations, the conclusion that A is basic for B follows. The current task facing us is that the component A is given, and we want to find a pair of B and C such that condition A is basic for B. The attention is thus about B and C, given A. We want to find B and C satisfying the required properties and inter relations with A. It is this general theme that guides us in the deployment of notations in the actual workouts.

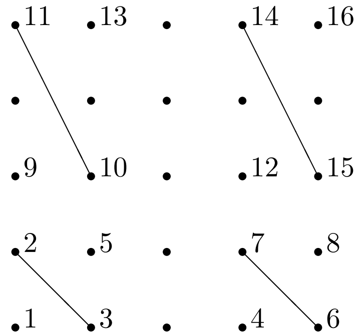
The condition we take up here to explain our approach is Figure 3, which is on a 4 by 4 product (see §5.1), and let us call it A. For each mounting of A on a 5 by 5 product, there is a unique row number, i, (counting from top to bottom), and a unique column number j (counting from left to right) that A does not use. Let Aij denote A mounted away from row i and column j. There are five i and j choices, resulting in 25 different cancellation conditions on the 5 by 5 product. The task on hand is to show that every Aij is basic for some array Bij. So, for each given Aij we want to uncover a (Bij, Cij) pair meeting the required properties and inter relations set forth in Theorem 16.

The inter relations between the three components can be observed visually without calculations. The relation which ties Bij and Cij is tight. We can figure out what Bij is once we know Cij, and vice versa. So, for the pair (Bij, Cij), to save space, we need to exhibit just one of the two, and we choose the latter Cij because there is a required property of Cij which is not visually obvious – additivity. That is where a worksheet is called upon. We use a worksheet that determines additivity, and start finding Cij with that worksheet. Once an additive Cij is found, the proof that Aij is basic for some Bij is done. When all (25) Cij are successfully found, the worksheet is saved as evidence of a proof. Neither the Aij and the resulting Bij are displayed in the worksheet. Near the end of the worksheet, codes are provided to bring an array to its canonical form. The canonical form of each Cij is visually doubled checked. In particular, if the highest value at the upper right corner is less than 21 then an error has occurred. That worksheet has filename "additivity5by5(proof-4by4-Fig3-embedded-and-checked).mw". The filename indicates which worksheet is used, which figure is embedded, and that a double check facility is included.

We pick A23 to illustrate fully the steps taken. The picture for A23 is shown here.



A23



B23, partially filled

To begin the creation of the B23 and C23 a reasonable starting point is to go back to the source of A. A linear critical-to-inspect 4×4 array should have been reported for which A is basic. In §5.3 it is found and is super-imposed on Fig 3. Bring that 4×4 array into A23, in exactly the same way A is mounted, and remove the $\oplus$ and $\oplus$ signs. The result is a partially filled B23, as shown. In concept, B23 refers to the array alone, and does not include the lines of comparisons. The lines of comparison are still left there as auxiliary indicators, because the super-imposed mode allows for future visual checking that A23 is violated when B23 is fully filled.

Next we "equalize" the compared entries of the partially filled B23, say, by lowering the higher ends and obtain a partially filled C23.

10	13	•	14	16
•	•	•	•	•
9	10	•	12	14
2	5	•	6	8
1	2	•	4	6

C23, partially filled

10	13	13.5	14	16
9.5	11.6	12.6	12.7	15.7
9	10	11.5	12	14
2	5	5.5	6	8
1	2	3.5	4	6

C23, additive

We enter this partially filled 5 by 5 array in a worksheet which performs the function – determine if an array is additive. On that worksheet we start completing the array C23. We fill the blank row 2 and column 3 with new numeric values (i.e., different from the integers 1 to 16). In so doing, maintain the required properties: (1) the values in rows and columns are increasing, (2) no same newly introduced values are repeated, and (3) values intercepting the compared ends in the partially filled B23 are not used, i.e., do not use decimals like $n.x$ where $n = 2, 6, 10, 14$ (the four equalized lower end values). When an additive C23 is found the proof is considered done. As a follow up (not shown in the worksheet) we copy the newly found row 2 and column 3 of C23 entries into the partially filled B23, and complete the B23 (and that is the tight inter relation between B23 and C23) explicitly. A23 is then basic for B23. A pair of B23 and C23 so found is displayed, with B23 super-imposed on A23.

•11	•13	•13.5	•14	•16
•9.5	•11.6	•12.6	•12.7	•15.7
•9	•10	•11.5	•12	•15
•2	•5	•5.5	•7	•8
•1	•3	•3.5	•4	•6

B23, from C23

•14	•20	•21	•22	•25
•12	•16	•18	•19	•24
•11	•13	•15	•17	•23
•2	•6	•7	•9	•10
•1	•3	•4	•5	•8

B23, in canonical form, basic

We leave a collection of (12) saved worksheets in [6], in pdf, and in ASCII (see Dataset) as the proof that all figures in Part 2 are basic for S_2.

4. $\mathcal{C}(S)$ where S is the class of all non-additive linear orders on a product X with given size (m, n) between $(3, 3)$ and $(5, 5)$

THEOREM 4.1. *Let S be the class of all non-additive linear orders on a product X whose size is at least 3 by 3 and below 5 by 5. Then $\mathcal{C}(S)$ is the disjoint union of two parts. Part 1 is the family of all basic figures reported* *on X for the non-additive linear critical-to-inspect subclass of S. Part 2 consists of all the basic figures known for smaller size products, mounted on (or embedded in) all possible sub-products of X. All conditions are basic for S.*

PROOF. The prelude posted before Theorem 3.1 has a leading paragraph which is not size specific. Following that lead we see that the family $\mathcal{C}(S)$ is adequately large – that it is violated by every non-additive linear order on X. The Part 1 figures are clearly basic for S. The fact that the Part 2 figures are basic for S is a simple consequence of the proven corresponding statement in Theorem 3.1 about 5 by 5 products. The argument is delivered through the following Lemma 4.2. Prior to applying the Lemma, we identify X with a sub-product of a 5 by 5 product Y. So the pair (X, Y) here has the role of the pair (W, X) stated in the Lemma. The Lemma, in loose terms, asserts that if a figure is basic for a larger product, then it is basic for a smaller product, provided that the figure is held within the smaller product. $\square$

LEMMA 4.2. *Let X be a finite two-dimensional product, and let $W \subset X$ be a sub-product (of size at least 3 by 3, say). If a cancellation sequence (x^1, y^1), $(x^2, y^2), \ldots, (x^J, y^J)$ in W is basic for the class of all non-additive linear orders on X then it is basic for the class of all non-additive linear orders on W.*

PROOF. Let (x^1, y^1), (x^2, y^2), $\ldots$, (x^J, y^J) be a given cancellation sequence in W and suppose that it is basic for the class of all (non-additive, understood) linear orders on X.

Then there exists a linear order $\succ$ on X such that (1) $x^1 \succ y^1, x^2 \succ y^2, \ldots, x^J \succ y^J$ and (2) for all cancellation sequence $(\tilde{x}^1, \tilde{y}^1)$, $(\tilde{x}^2, \tilde{y}^2)$, $\ldots$, $(\tilde{x}^K, \tilde{y}^K)$ in X, $\tilde{x}^1 \succ \tilde{y}^1$, $\tilde{x}^2 \succ \tilde{y}^2, \ldots, \tilde{x}^K \succ \tilde{y}^K$ implies (the set) $\{(\tilde{x}^1, \tilde{y}^1), (\tilde{x}^2, \tilde{y}^2), \ldots, (\tilde{x}^K, \tilde{y}^K)\}$ contains $\{(x^1, y^1), (x^2, y^2), \ldots, (x^J, y^J)\}$.

By restriction from X to W, property (2) clearly implies property (3): for all cancellation sequence $(\tilde{x}^1, \tilde{y}^1)$, $(\tilde{x}^2, \tilde{y}^2)$, $\ldots$, $(\tilde{x}^K, \tilde{y}^K)$ in W, $\tilde{x}^1 \succ \tilde{y}^1$, $\tilde{x}^2 \succ \tilde{y}^2$, $\ldots$, $\tilde{x}^K \succ \tilde{y}^K$ implies $\{(\tilde{x}^1, \tilde{y}^1), (\tilde{x}^2, \tilde{y}^2), \ldots, (\tilde{x}^K, \tilde{y}^K)\}$ contains $\{(x^1, y^1), (x^2, y^2), \ldots, (x^J, y^J)\}$.

Let $\succ_W$ be the restriction of $\succ$ to W. Then $\succ_W$ is a linear order on W. Replacing in properties (1) and (3) $\succ$ by the new name $\succ_W$, we have shown that (x^1, y^1), (x^2, y^2), $\ldots$, (x^J, y^J) is basic for $\succ_W$. So it is basic for the class of all linear orders on W. $\square$

REMARK. Theorem 4.1 here reports on products of sizes below $(5, 5)$. The two former papers, [4] and [5], also reports on products of sizes below $(5, 5)$. What is new? The assertion that $\mathcal{C}(S)$ is adequately large is not new. The assertion that $C(S)$ is the simplest – that all part 2 figures are basic and not a single one can be removed – is new. Fishburn's question in [2] is about the maximum width of the sequences in $\mathcal{C}(S)$. As we see, no matter where a figure in part 2 is embedded, the width does not vary. Moreover, only the maximum width of all figures need be told. So we did not give the question "Is the family the simplest?" full attention. The previous papers cover also all non-linear weak orders. The current work is narrower in scope. Bringing those results on non-linear orders to the scene, we can strengthen Theorem 4.1. In fact, enlarging the family from the linear ones to all weak orders, the family $\mathcal{C}(S)$ stays unchanged and of course, stays being the simplest.

It is a conjecture that Theorem 3.1 can also be so strengthened. A very large sub-class of the class of all 5 by 5 non-linear critical-to-inspect arrays has been obtained and included in the repository ([6, Dataset]). All arrays in the sub-class violates the cancellation conditions stated in Part 1 – the (105) clusters. We are not done with examining the full class yet. The challenge is in getting the full class.

The paper [3] contains results for a product X whose size is $(3, 6)$. It is natural to ask if Theorem 4.1 can be extended to cover that size. The assertion in Part 2 – that all conditions are basic for S – needs be checked.

All worksheets used, and a few sample Maple sessions are provided in [6], intended for readers with access to Maple and wish to get a feel of the production of the data (e.g., Dataset).

The reference list in this paper is kept short. The survey by Slinko [7] is extensive and provides an excellent source of references.

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