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ON A CERTAIN POLISH MATHEMATICIAN: A BRIEF SUMMARY OF SCIENTIFIC RESEARCH AND ACHIEVEMENTS OF KAZIMIERZ NIKODEM, PROFESSOR OF MATHEMATICS

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Figure 1. Prof. Kazimierz Nikodem

Kazimierz Nikodem was born in 1953 in Wojkowice. He began his initial mathematical education at primary school and then secondary school in Wojkowice. He began his studies in mathematics in 1972 at the Faculty of

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Mathematics, Physics and Chemistry of the University of Silesia in Katowice. His master thesis was entitled "Wypukłe procesy stochastyczne" ("Convex Stochastic Processes") and written under the supervision of Professor Marek Kuczma. He defended it in 1977 obtaining a master's degree. Kazimierz Nikodem obtained a Doctoral Degree in Mathematical Sciences in 1981, defending his doctoral dissertation entitled "Wypukłe i kwadratowe procesy stochastyczne" ("Convex and Quadratic Stochastic Processes"), which was supervised by Professor Roman Ger. He obtained the post-doctoral degree (habilitation) in 1991. This title was awarded by the Polish Academy of Sciences in Warsaw, based on the defended post-doctoral thesis "K-convex and K-concave set-valued functions". Kazimierz Nikodem received the title of full professor from the President of the Republic of Poland Aleksander Kwaśniewski in 2001.

Privately, Kazimierz Nikodem is married to Jadwiga Nikodem, has two children and five grandchildren.

Kazimierz Nikodem began his professional career in 1977 as an assistant at the Institute of Mathematics of the University of Silesia. Since 1984, he has been employed at the Department of Mathematics at the University of Bielsko-Biala, being Head of the Department between 1997 and 2023.

Kazimierz Nikodem is the scientific father of four promoted doctors of mathematical sciences: Dr. hab. Szymon Wąsowicz, Dr. Elżbieta Sadowska-Owczorz, Dr. Mirosław Adamek and Dr. Dawid Kotrys.

Figure 2. Prof. Kazimierz Nikodem and his four PhD students: Dr. Mirosław Adamek, Prof. Szymon Wąsowicz, Dr. Elżbieta Sadowska-Owczorz, Dr. Dawid Kotrys

Main areas of Kazimierz Nikodem's research:

1. Convex and quadratic stochastic processes.
2. Set-valued solutions of functional equations.
3. Convex and concave set-valued maps.
4. Jensen-convex, Wright-convex and Schur-convex functions.
5. Hyers-Ulam stability.
6. Separation theorems and selections.
7. Strongly convex functions.
8. Ohlin Lemma - generalizations and applications.
9. Set-valued means.

Let us present some results obtained in the above areas.

Let $(\Omega, \mathcal{A}, P)$ be an arbitrary probability space. A function $X\colon D\times\Omega \to \mathbb{R}$, where $D \subset \mathbb{R}^n$, is called a stochastic process iff for all $t \in D$ the function $X(t,\cdot)\colon \Omega \to \mathbb{R}$ is a random variable.

A stochastic process $X\colon D \times \Omega \to \mathbb{R}$ is called

(i) P-upper bounded on a non-empty set $A \subset D$ iff

$$\lim_{n\to\infty} \sup_{t\in A} \{P(\{\omega \in \Omega : X(t,\omega) \geq n\})\} = 0,$$

(ii) P-lower bounded on a non-empty set $A \subset D$ iff

$$\lim_{n\to\infty} \sup_{t\in A} \{P(\{\omega \in \Omega : X(t,\omega) \leq -n\})\} = 0,$$

(iii) P-bounded on a non-empty set $A \subset D$ iff it is P-upper bounded and P-lower bounded on the set A,

(iv) midconvex iff

$$X\left(\frac{u+v}{2}, \cdot\right) \leq \frac{X(u,\cdot)+X(u,\cdot)}{2} \quad \text{(a.e.)}$$

for all $x, y \in D$, where $D \subset \mathbb{R}^n$ is an non-empty, open and convex set,

(v) quadratic iff

$$X(u+v,\cdot) + X(u-v,\cdot) = 2X(u,\cdot) + 2X(v,\cdot) \quad \text{(a.e.)}$$

for all $u, v \in D$ such that $u+v, u-v \in D$,

(vi) P-continuous in D iff for all $t_0 \in D$

$$P - \lim_{t\to t_0} X(t,\cdot) = X(t_0,\cdot),$$

where $P - \lim$ denotes the limit in probability.

Theorem 1 gives us a condition under which a stochastic process has to be P-continuous.

THEOREM 1 ([5, Theorem 4]). *Let $(\Omega, \mathcal{A}, P)$ be an arbitrary probability space and D be a convex open subset of $\mathbb{R}^n$. If a stochastic process $X\colon D\times\Omega \to \mathbb{R}$ is midconvex and P-upper bounded on a set with a non-empty interior, then it is P-continuous in D.*

Theorem 2 says that there is a decomposition of the space Ω into two sets, such that the quadratic stochastic process is midconvex and midconcave on each of them, resp.

THEOREM 2 ([6, Lemma 5]). *Let $(\Omega, \mathcal{A}, P)$ be an arbitrary probability space. If a stochastic process $X\colon \mathbb{R} \times \Omega \to \mathbb{R}$ is quadratic and P-bounded on a set with non-empty interior, then there exists a measurable subset Ω_1 of Ω such that X is midconvex on Ω_1 and midconcave on $\Omega \setminus \Omega_1$.*

Let X be a vector space and $n(X)$ denote the family of all non-empty subsets of X. Recall that set-valued map $F\colon D \to n(X)$, D being a convex subset of some vector space, satisfies Jensen functional equation iff

$$F\left(\frac{u+v}{2}\right) = \frac{F(u)+F(v)}{2},$$

for all $u, v \in D$.

In Theorem 3 we have a characterization of the solutions of Jensen functional equation for set-valued maps with compact values in a topological vector space.

THEOREM 3 ([8, Theorem 2]). *Let X be a locally convex topological vector space and $c(X)$ denote the family of all compact non-empty subsets of X. A set-valued map $F\colon [0,\infty) \to c(X)$ satisfies the Jensen functional equation if and only if $F(s) = a(s)+sA+B$, $s \in [0,\infty)$, where $a\colon \mathbb{R} \to X$ is an additive function and A, B are compact convex subsets of X.*

A set-valued map $F\colon D \to n(Y)$, D being a convex subset of a vector space X, is said to be K-midconvex, K is a cone in X, iff

$$\frac{F(u)+F(v)}{2} \subset F\left(\frac{u+v}{2}\right) + K,$$

for all $u, v \in D$.

Theorem 4 gives us a condition under which a K-midconvex set-valued map is K-continuous. K-continuous at a point $x_0 \in D$ means that for every neighborhood W of zero in Y there exists a neighborhood U of zero in X such that

$$F(x_0) \subset F(x) + W + K$$

and

$$F(x) \subset F(x_0) + W + K$$

for every $x \in (x_0 + U) \cap D$.

THEOREM 4 ([9, Theorem 3.8]). *Let X be a real Polish space and Y be a topological vector space. Assume that $D \subset X$ is an open convex set, $A \subset D$ has non-empty interior, K is a convex cone in Y and $B(Y)$ is the family of all bounded subsets of Y. If a set-valued map $F\colon D \to B(Y)$ is K-midconvex and $G\colon D \to B(Y)$ is a Christensen weakly measurable set-valued map such that $G(x) \subset F(x) + K$, $x \in A$, then F is K-continuous on D.*

A function $f\colon D \to \mathbb{R}$ is called midconvex iff

$$f\left(\frac{x+y}{2}\right) \leq \frac{f(x)+f(y)}{2}$$

for all $x, y \in D$; it is called midconcave if it satisfies the reverse inequality.

THEOREM 5 ([7, Theorem 1]). *Assume that D is an open and convex subset of $\mathbb{R}^n$. If a midconvex function $f_1\colon D \to \mathbb{R}$ is majorized on D by a midconcave function $f_2\colon D \to \mathbb{R}$, then there exist an additive function $a\colon \mathbb{R}^n \to \mathbb{R}$, a convex function $g_1\colon D \to \mathbb{R}$ and a concave function $g_2\colon D \to \mathbb{R}$ such that $f_1(x) = a(x) + g_1(x)$ and $f_2(x) = a(x) + g_2(x)$, for all $x \in D$.*

Let Y be a real vector space and K a proper convex cone in Y. Denote by (Y, K) the (partially) ordered vector space with the order structure defined by $x \leq y$ iff $y - x \in K$. The space (Y, K) is said to be order complete if every non-empty subset B of Y that is bounded above has the least upper bound, $\sup B$. Then every non-empty set $B \subset Y$ that is bounded below has the greatest lower bound, $\inf B$.

Theorem 6 gives us a characterization of midconvex mappings with values in a vector space.

THEOREM 6 ([10, Theorem 2]). *Let X be a real vector space, D a convex algebraically open subset of X and (Y, K) an order complete ordered vector space. A mapping $f\colon D \to Y$ is midconvex if and only if it has Jensen support at every point of D.*

A function $f\colon D \to \mathbb{R}$ is called t-Wright-convex, t is a fixed number, iff

$$f(tx + (1-t)y) + f((1-t)x + ty) \le f(x) + f(y)$$

for all $x, y \in D$.

THEOREM 7 ([2, Corollary]). *Let D be a convex subset of a real vector space. If a function $f\colon D \to \mathbb{R}$ is t-Wright-convex with a rational $t \in (0,1)$, then it is midconvex.*

The next theorem addresses the problem of stability of midconvex functions.

A function $f\colon D \to \mathbb{R}$ is called

(i) ε-midconvex iff

$$f\left(\frac{x+y}{2}\right) \le \frac{f(x)+f(y)}{2} + \varepsilon$$

for all $x, y \in D$,

(ii) ε-convex iff

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y) + \varepsilon$$

for all $x, y \in D$ and $t \in (0,1)$.

THEOREM 8 ([12, Theorem 1]). *Let X be a real vector space and $D \subset X$ be open convex. If $f\colon D \to \mathbb{R}$ is ε-midconvex and locally bounded from above at a point of D, then f is 2ε-convex.*

Theorem 9 gives us a necessary and sufficient condition for separating two functions by a convex function.

THEOREM 9 ([1, Theorem 1]). *Let $I \subset \mathbb{R}$ be an interval. Functions $f, g\colon I \to \mathbb{R}$ satisfy*

$$f(tx + (1-t)y) \le tg(x) + (1-t)g(y), \quad x, y \in I,\ t \in [0,1],$$

if and only if there exists a convex function $h\colon I \to \mathbb{R}$ such that

$$f \le h \le g.$$

Theorem 10 gives us a necessary and sufficient condition for separating two functions by an affine function.

Theorem 10 ([15, Theorem 1]). *Let $I \subset \mathbb{R}$ be an interval and $f, g\colon I \to \mathbb{R}$. There exist an affine function $h\colon \mathbb{R} \to \mathbb{R}$ such that $f \le h \le g$ on I if and only if*

$$f(tx + (1-t)y) \le tg(x) + (1-t)g(y)$$

and

$$g(tx + (1-t)y) \ge tf(x) + (1-t)f(y)$$

for all $x, y \in I$ and $t \in [0,1]$.

Let $(X, \|\cdot\|)$ be a real normed space. A function $f\colon D \to \mathbb{R}$ is called strongly convex with modulus $c > 0$ iff

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y) - ct(1-t)\|x-y\|^2$$

for all $x, y \in I$ and $t \in [0,1]$.

In Theorem 11 we can find a Hermite-Hadamard type inequality for strongly convex functions.

Theorem 11 ([3, Theorem 6]). *If a function $f\colon I \to \mathbb{R}$ is strongly convex with modulus c then*

$$f\left(\frac{a+b}{2}\right) + \frac{c}{12}(b-a)^2 \le \frac{1}{b-a}\int_a^b f(x)\,dx \le \frac{f(a)+f(b)}{2} - \frac{c}{6}(b-a)^2,$$

for all $a, b \in I$, $a < b$.

Theorem 12 gives us a characterization of inner product spaces among normed spaces involving the notion of strong convexity.

Theorem 12 ([13, Theorem 2.3]). *Let $(X, \|\cdot\|)$ be a real normed space. The following conditions are equivalent to each other:*

1. *For all $c > 0$ and for all functions $f\colon D \to \mathbb{R}$, f is strongly convex with modulus c if and only if $g = f - c\|\cdot\|^2$ is convex.*
2. *There exists a $c > 0$ such that, for all functions $g\colon D \to \mathbb{R}$, g is convex if and only if $f = g + c\|\cdot\|^2$ is strongly convex with modulus c.*
3. *$\|\cdot\|^2\colon X \to \mathbb{R}$ is strongly convex with modulus 1.*
4. *$(X, \|\cdot\|)$ is an inner product space.*

In Theorem 13 we have a characterization of strongly Wright-convex functions.

A function $f\colon D \to \mathbb{R}$ is called strongly Wright convex with modulus $c > 0$ iff

$$f(tx + (1-t)y) + f((1-t)x + ty) \le f(x) + f(y) - 2ct(1-t)\|x-y\|^2$$

for all $x, y \in D$.

Theorem 13 ([4, Corollary 5]). *Let $(X, \|\cdot\|)$ be a real inner product space, D be an open convex subset of X and $c > 0$. A function $f\colon D \to \mathbb{R}$ is strongly Wright-convex with modulus c if and only if there exist a convex function $g\colon D \to \mathbb{R}$ and an additive function $a\colon X \to \mathbb{R}$ such that*

$$f(x) = g(x) + a(x) + c\|x\|^2, \quad x \in D.$$

In Theorem 14 the authors present an Ohlin type result for strongly convex functions.

Theorem 14 ([14, Theorem 2]). *Let $X, Y\colon \Omega \to I$ be square integrable random variables such that $\mathbb{E}[X] = \mathbb{E}[Y]$. If there exists a $t_0 \in \mathbb{R}$ such that*

$$F_X(t) \le F_Y(t) \quad \text{if} \quad t < t_0 \quad \text{and} \quad F_X(t) \ge F_Y(t) \quad \text{if} \quad t > t_0,$$

then

$$\mathbb{E}[f(X)] - c\mathbb{D}^2[X] \le \mathbb{E}[f(Y)] - c\mathbb{D}^2[Y]$$

for every continuous function $f\colon I \to \mathbb{R}$ strongly convex with modulus c.

The last theorem applies to multivalued means.

Theorem 15 ([11, Theorem 2.2]). *Let $f, g\colon I \to J$ be continuous and strictly increasing (strictly decreasing) functions. Assume that $f \le g$ on I and $F(x) = [f(x), g(x)]$, $x \in I$. Then, the following conditions are equivalent:*

1. *For every $n \ge 2$, the map $A_F\colon I^n \to S(I)$ given by*

$$A_F(x_1, \ldots, x_n) = F^+\Big(\frac{1}{n}\sum_{i=1}^{n} F(x_i)\Big), \quad x_1, \ldots, x_n \in I,$$

   is a set-valued mean.

2. *The map* $A_F\colon I^2 \to S(I)$ *given by*

$$A_F(x_1,x_2) = F^+\Big(\frac{F(x_1)+F(x_2)}{2}\Big), \quad x_1,x_2 \in I,$$

is a set-valued mean.

3. *The function* $g \circ f^{-1}$ *is convex.*

Kazimierz Nikodem is the author/co-author of over one hundred scientific publications. Most of them have been published in scientific journals from the ISI Master Journal List (in Polish: lista filadelfijska). His research and the results obtained have often been, and still are, an inspiration for further scientific research conducted by mathematicians not only from Poland, but almost all over the world. His enormous influence on the broad mathematical community is evidenced by the number and diversity of his co-authors, the number of citations of his papers and scientific visits to many foreign universities.

Co-authors of scientific papers: Poland: Karol Baron (1), Roman Ger (2), Janusz Matkowski (4), Eliza Jabłońska (3), Witold Jarczyk (1), Mirosław Adamek (5), Dawid Kotrys (4), Jadwiga Nikodem (1), Joanna Nikodem (1), Mateusz Nikodem (1), Teresa Rajba (5), Elżbieta Sadowska (1), Szymon Wąsowicz (7); Hungary: Zsolt Páles (18), Attila Gilányi (5), Gyula Maksa (1), Judit Makó (1); Venezuela: Nelson Merentes (10), José Luis Sánchez (3), Antonio Azocar (2), Sergio Rivas (2), Hugo Leiva (1), Teodoro Lara (1), José Gimenez (2), Carlos González (2), Luisa Sánchez (1), Gari Roa (2), Lysis González (1), Odalis Mejia (1), Hiliana Angulo (1), Ana Milena Moros (1); Romania: Dorian Popa (4), Gábor Kassay (1), Mircea Balaj (1), Flavia-Corina Mitroi (1); Italy: Francesca Papalini (3), Tiziana Cardinali (1), Antonella Fiacca (1), Susanna Vercillo (1); China: Weinian Zhang (3), Bing Xu (2); Croatia: Milica Klaričić Bakula (4); Morocco: Iz-iddine El-Fassi (3); Australia: Silvestru Sever Dragomir (2); USA: Thomas Riedel (1), Prasanna Sahoo (1); Canada: Che Tat Ng (1); Austria: Wolfgang Förg-Rob (1); Germany: Ehrhard Behrends (1); Iran: Mohammad Sal Moslehian (1), Hamid Reza Moradi (1), Mohsen Erfanian Omidvar (1); Pakistan: Muhammad Adil Khan (1); Saudi Arabia: El-Sayed El-Hady (1).

Citations:

- total number of citations according to Web of Science – 758, according to Google Scholar – 2368,
- Hirsch Index according to Web of Science – 15, according to Google Scholar – 27,
- Index i-10 according to Google Scholar – 54.

Scientific visits to universities: Waterloo (1988), Perugia (1992, 1993), Caracas (1996, 2006, 2010, 2011, 2013, 2014, 2015), Debrecen (1997, 2002,

Figure 3. Prof. Kazimierz Nikodem with students in Tabriz

2005, 2008, 2010, 2013), Split (2015, 2016, 2017), Innsbruck (1998), Cluj-Napoca (2000, 2019), Chengdu (2000, 2006, 2015, 2017), Oradea (2005), Louisville (2006), Hong Kong (2006), Merida (2011), Chongquing (2015), Neijiang (2015), Melbourne (2016), Tokyo-Chiba (2016), Jiaxing (2018), Hanoi (2018).

His scientific and teaching work has been noticed and appreciated many times, which is reflected not only in the information presented above, but also in the awards, distinctions and decorations received.

Prizes:

– 3rd Prize in the Józef Marcinkiewicz Competition for the best student work (1977)
– 3rd degree individual award of the Minister of National Education for scientific achievements (1988)
– Awards in the Marek Kuczma Competition for the best Polish scientific work on functional equations (1st prize for 1989, 1st prize for 1998, 3rd prize for 1981)
– Award of the Rector of the University of Silesia (1982)
– Over 20 awards of the Vice-Rector of the Lodz University of Technology in Bielsko–Biala and the Rector of the University of Bielsko–Biala (in the years 1984-2022)

Distinctions and decorations:

– Medal of the National Education Commission (1999)
– Silver Cross of Merit (1998)
– Golden Cross of Merit (2003)

– PRIMUS INTER PARES Nicolas Copernicus Silver Distinction (1977)
– Honorary Badge “For merits for the Bielsko Voivodeship” (1998)
– Golden Honorary Badge “For merits for the Silesian Voivodeship” (2014)
– Honorary Badge of Merit for the University of Bielsko–Biala (2019)

Up to the present day Kazimierz Nikodem remains an active scientist and academic teacher who willingly shares his experience and knowledge with his students and colleagues, thus supporting them in scientific research.

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