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Frege's Way Out
Stanisław Leśniewski's Unpublished Archival Document (1938)

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Abstract: This paper presents the transcription of a previously unknown archival document: Stanisław Leśniewski, *The Basic Scheme of a Proof for the Theorem* "*a = b*" *in the Reformed Frege's System* [1938], written in German and stored in one of Berlin's archives. It was sent to Heinrich Scholz, professor at the University of Münster (Germany). Nevertheless, it is kept in the legacy of Karl Schröter, who was previously Scholz's student and co-worker. Furthermore, we provide introductory remarks on the problem of Russell's antinomy in Gottlob Frege's logical system from *Basic Laws of Arithmetic*, its subsequent correction, and the origin of the term *Frege's way out*. We conclude that the document *Basic Scheme of a Proof* sheds new light on Frege's logical system correction given by him in the *Afterword* to the second volume of the book.

Keywords: *Frege's way out*, Stanisław Leśniewski, Russell's antinomy

Introduction

For the first time, we publish the following transcribed archival document authored by Stanisław Leśniewski: *The Basic Scheme of a Proof for the Theorem "a = b" in the Reformed Frege's System*.[1] Leśniewski wrote it in May 1938 and sheds new light on his last research on the problem of antinomy in Frege's logic. At the beginning, we give some introductory remarks on the problem of Russell's antinomy in Gottlob Frege's logical system from *Basic Laws of Arithmetic*,[2] its correction and how the name *Frege's way out* was coined.

Preliminary Remarks

Leśniewski's two-page, handwritten German document, *The Basic Scheme of a Proof for the Theorem "a = b" in the Reformed Frege's System*, signed by him, is likely his final text on the antinomy in Gottlob Frege's corrected logical system, which was presented in the *Afterword* to the two-volume *Basic Laws of Arithmetic*.[3] The document was sent to Heinrich Scholz, professor of philosophy at the University of Münster, on December 7, 1938.[4]

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[1] The original document entitled *Das Grundschema eines Beweises für den Satz "a = b" im reformierten Fregeschen System* is stored in Akademiearchiv. Berlin-Brandenburgische Akademie der Wissenchaften (Berlin, Germany), Nachlass Karl Schröter, Bestand 2. We would like to thank the archive for allowing us to publish the transcribed document. For more historical details about the document and its abbreviation in natural language see G. Besler, R. Miszczyński: "Stanisław Leśniewski's Collaboration with Heinrich Scholz and His School." *History and Philosophy of Logic* (2025) (forthcoming). Translation of the title of the original document and other passages in natural language, from German into English, by Philip A. Ebert (University of Stirling), in consultation with Marcus Rossberg (University of Connecticut), to both of whom we are grateful.

[2] G. Frege: *Basic Laws of Arithmetic. Derived Using Concept-script*. Translated and edited Ph. A. Ebert, M. Rossberg with C. Wright. And the advice of M. Beaney et al. Appendix by R. T. Cook. Vol. 1–2. Oxford University Press, Oxford 2013.

[3] G. Frege: "Afterword." In: Idem: *Basic Laws of Arithmetic…*, vol. 2, pp. 251–262.

[4] H. Scholz: *Sprechen und Denken. Ein Bericht über neue gemeinsame Ziele der polnischen und der deutschen Grundlagenforschung*. Mianowski Institute for the Promotion of

Scholz's collaboration with Polish logicians is well known. He visited Poland twice, in 1932 and 1938. Logicians from the Lviv-Warsaw School also stayed in Münster. On April 23–24, 1938, Leśniewski delivered two lectures there, focusing on his ontology and the topic later known as *Frege's way out*. According to Heinrich Scholz, the document was written shortly after Leśniewski's return to Warsaw.[5] There, Leśniewski used Frege's terminology and theorems; however, he employed the symbolism of *Principia Mathematica*.[6] Leśniewski proved that even the corrected Frege's logical system leads to Russell's antinomy, provided there is more than one element in the universe.

Although Leśniewski addressed the problem of Russell's antinomy for over 25 years, he did not begin working on the topic at the outset of his research. His philosophical attitude is usually called 'intuitive formalism.' Nevertheless, the name is irrelevant to his first logical publications, including *A Contribution to the Analysis of Existential Propositions*[7] (1911), which was based on his PhD thesis written under Kazimierz Twardowski's supervision and defended in 1912 in Lviv. In this period, Leśniewski was acquainted only with John Stuart Mill's logic and knew nothing about the new possibilities of doing logic; he used natural language instead of symbolic one. After a quarter of a century, he referred to his early papers as one-sided 'philosophical' – grammatical culture.[8] His attitude was formed by "the problems of 'universal-grammar' and of logico-semantics in the style of Edward [sic! Edmund] Husserl and by the exponents of the so-called Austrian School."[9] In this period, Leśniewski was not interested in the problem of antinomy yet.

Science and Letters in Poland, Warsaw 1939, p. 4. Polish translation: Idem: "Mowa i myślenie. Komunikat o nowych wspólnych celach badań podstawowych prowadzonych w Polsce i Niemczech." In: *Fenomen Szkoły Lwowsko-Warszawskiej*. Red. A. Brożek, A. Chybińska. Wydawnictwo Academicon, Lublin 2016, pp. 191–221.

[5] Ibidem.

[6] A. N. Whitehead, B. Russell: *Principia Mathematica*. Vol. 1–3. Cambridge University Press, Cambridge 1910–1913.

[7] S. Leśniewski: *A Contribution to the Analysis of Existential Propositions*. In: Idem: *Collected Works*. Eds. S. J. Surma, J. T. Srzednicki, D. I. Barnett, with an annotated bibliography by V. F. Rickey. Vol. 1–2. Kluwer Academic Publishers–PWN-Polish Scientific Publish, Dordrecht–Boston–Warszawa 1992, vol. 1, pp. 1–19. First publication: "Przyczynek do analizy zdań egzystencjalnych." *Przegląd Filozoficzny*, 14 (1911), pp. 329–345.

[8] S. Leśniewski: *On the Foundations of Mathematics*. [Introduction, Chapters I–III]. In: Idem: *Collected Works*…, vol. 1, p. 197.

[9] Ibidem, p. 181.

American scholar Vito F. Sinisi[10] pointed out three analyses of Russell's antinomy conducted by Leśniewski:

The first, from 1914 appeared in Leśniewski's paper, *Is the class of classes which are not elements of themselves an element of itself?*, where he presented a critical analysis of the antinomy.[11]

The second, from 1927, is presented in the second chapter of *On the Foundations of Mathematics*, an incomplete series of articles on the foundations of mathematics.[12]

The third, from 1938, was reconstructed by Leśniewski's student, Bolesław Sobociński, in a paper *L'analyse de l'antinomie russellienne par Leśniewski*,[13] published 12 years after Leśniewski's death, based on Sobociński's memory.

The published document *Das Grundschema eines Beweises*, belongs to Leśniewski's final period. A study of the document reveals the long journey Leśniewski took from his early works on Russell's antinomy to the conclusions presented in the document sent to Scholz in 1938.

Before briefly describing the three formulations of Russell's Antinomy, we outline the main idea of Russell's antinomy and *Frege's way out*.

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[10] V. F. Sinisi: "Leśniewski's Analysis of Russell's Antinomy." *Notre Dame Journal of Formal Logic*, 17/1 (1976), pp. 16–34; comp. R. Miszczyński: "Gottlob Frege a szkoła lwowsko-warszawska. Problem 'Frege's way out'." In: *Filozofia polska na tle filozofii europejskiej w XX wieku*. Ed. M. Woźniczka. Akademia im. Jana Długosza w Częstochowie, Częstochowa 2014, pp. 211–220; R. Miszczyński: "Stanisława Leśniewskiego drugie rozwiązanie antynomii Russella." *Prace Naukowe Akademii im. Jana Długosza w Częstochowie. Seria: Filozofia*, 8 (2011), pp. 163–172; Idem: "Stanisława Leśniewskiego pierwsze rozwiązanie antynomii Russella." *Prace Naukowe Akademii im. Jana Długosza w Częstochowie. Seria: Filozofia*, 7 (2010), pp. 5–17; Idem: "Stanisława Leśniewskiego trzecia analiza antynomii Russella." *Prace Naukowe Akademii im. Jana Długosza w Częstochowie. Seria: Filozofia*, 10 (2013), pp. 163–181.

[11] S. Leśniewski: *Is the Class of Classes not Subordinated to Themselves?*. Transl. S. J. Surma, J. Wójcik. In: S. Leśniewski: *Collected Works*…, vol. 1, pp. 115–128. First publication: "Czy klasa klas, nie podporządkowanych sobie, jest podporządkowana sobie?." *Przegląd Filozoficzny*, 17 (1914), pp. 63–75.

[12] S. Leśniewski: *On the Foundations of Mathematics*. [Introduction, Chapters I–III]…, pp. 174–226. First publication: "O podstawach matematyki: Wstęp i rozdziały I–III." *Przegląd Filozoficzny*, 30 (1927), pp. 164–206.

[13] B. Sobociński: "Leśniewski's Analysis of Russell's Paradox." Transl. R. E. Clay. In: *Leśniewski's Systems. Ontology and Mereology*. Eds. J. T. J. Srzednicki, V. F. Rickey. Ossolineum, Wrocław 1984, pp. 11–44.

Russell's antinomy in Gottlob Frege: *Basic Laws of Arithmetic*

In his first letter to Frege, dated 22 June 1902, Russell referred to the chapter *Function* from Frege's first book, *Conceptual Notation*,[14] where the difference between function and variable was introduced. Based on this chapter, rather than any chapter from *Basic Laws of Arithmetic*, Russell identified a contradiction: "I have encountered a difficulty only at one point. You assert […] that a function could also constitute the indefinite element. This is what I used to believe, but this view now seems to me dubious because of the following contradiction. Let *w* be the predicate of being a predicate which cannot be predicated of itself. Can *w* be predicated of itself? From either answer follows its contradictory."[15]

Here Russell referred to the following words from Frege's *Conceptual Notation*: "If, in an expression (whose content needs not be assertible), a simply or a complex symbol occurs in one or more places and we imagine it as replaceable by another [symbol] (but the same one each time) in all or some of these places, then we call the part of expression that shows itself invariant [under such replacement] a function and the replaceable part its argument. It can also happen that, conversely, the argument is determinate, but the function is indeterminate."[16]

Next, the difficulty that Frege noticed pertained to his logical system as presented in the first volume of *Basic Laws of Arithmetic*, whose second volume, after ten years, was about to be published. He quickly recognized that the problem stemmed from Basic Law V. To sum up, we emphasize that Frege himself identified an antinomy in *Basic Laws of Arithmetic* following Russell's brilliant remark regarding *Begriffsschrift*.

Unable to alter the content of the already printed first volume, Frege included an *Afterword*[17] at the end of the second volume, in which he presented Russell's construction and proposed a way to prevent it. His idea was to weaken Basic Law V by limiting the domain of its functions. He believed

[14] G. Frege: *Conceptual Notation: A Formula Language of Pure Thought Modelled upon the Formula Language of Arithmetic*. In: Idem: *Conceptual Notation and Related Articles*. Ed., transl., bibliogr., introd. T. W. Bynum. Clarendon Press, Oxford 1972, pp. 101–203.

[15] G. Frege: *Philosophical and Mathematical Correspondence*. Eds. G. Gabriel et al. Abridged for the English ed. B. McGuinness. Trans. H. Kaal. Basil Blackwell, Oxford 1980, p. 130.

[16] G. Frege: *Conceptual Notation*…, p. 127.

[17] G. Frege: *Afterword*…

this solution would eliminate the contradiction without requiring significant changes to the proofs in the published two-volume book. He believed that the reader's awareness of the need to make minor adjustments to the proofs presented in the works would save them from probable disqualification. Leśniewski himself was skeptical about Frege's attempt to eliminate the possibility of constructing an antinomy. He treated it as an *ad hoc* recipe, not based on proper intuitive foundations.

The first formulation of Russell's antinomy (1914)

Leśniewski first encountered the new logic in 1911 while reading Jan Łukasiewicz's book *The Principle of Contradiction in Aristotle*,[18] which had a profound impact on him as a young scholar. Later, he admitted: "This book […] became a revelation for me in many respects and for the first time in my life I learned of the existence of the 'symbolic logic' of Bertrand Russell as well as his 'antinomy' regarding the 'class of classes which are not elements of themselves'."[19] Leśniewski quickly began analyzing contradictions, searching for a way to avoid them. These can already be seen in *The Critique of the Logical Principle of the Excluded Middle*.[20] Not much later, in 1914, in the paper *Is the Class of Classes not Subordinated to Themselves*[21] he sketched the first version of a mereological solution to the problem of antinomy.

From Łukasiewicz's book, Leśniewski learned not only about Russell's contradiction. The book ends with an appendix.[22] In its very first sentence, Łukasiewicz informed the reader that only a few people were capable of

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[18] J. Łukasiewicz: *The Principle of Contradiction in Aristotle: A Critical Study.* Trans. and commentary H. Heine. Foreword G. Priest. Topos Productions, [s.l.] 2021. First publication: *O zasadzie sprzeczności u Arystotelesa. Studyum krytyczne*. Akademia Umiejętności, Kraków 1910.

[19] S. Leśniewski: *On the Foundations of Mathematics…*, p. 181.

[20] S. Leśniewski: *The Critique of the Logical Principle of the Excluded Middle.* Transl. S. J. Surma and J. Wójcik. In: Idem: *Collected Works.* Vol. 1…, pp. 47–85. First publication: "Krytyka logicznej zasady wyłączonego środk[a]." *Przegląd Filozoficzny*, 16 (1913), pp. 315–352.

[21] S. Leśniewski: *Is the Class of Classes not Subordinated to Themselves?…*

[22] J. Łukasiewicz: *Appendix I: The Principle of Contradiction and Symbolic Logic*. In: Idem: *The Principle of Contradiction…*, pp. 160–191.

treating traditional logic in a novel manner. Leśniewski was not among them. Unfortunately, he did not acquire the relevant competencies during his educational journey across Europe before arriving in Lviv. Moreover, preparing for the doctoral exam in mathematics was probably not a sufficient prerequisite for effectively mastering the new logical techniques.[23]

The young doctor's shortcomings caused significant difficulties when he attempted to understand the discussions on contradiction conducted in the artificial formal language of *Principia Mathematica*. Leśniewski did not accept this form of communication and stated that he "was not able to grasp the real 'sense' of the axioms and theorems of that theory – of what' and 'what', respectively, it was desired to 'assert' by means of the axioms and theorems".[24]

However, Leśniewski's initial difficulties in understanding content expressed through artificial logical signs – which he eventually overcame – were not solely due to his insufficient proficiency in formal language. He indicated that they were instead related to his tendency to interpret Whitehead and Russell's comments literally. Leśniewski strongly criticized them, accusing the authors of intellectual sadism toward the potential readers of the offered explanations. He rhetorically asked to what extent the aforementioned remarks exhibit a subtle cruelty toward a reader who is used to thoughtfully analyzing what he reads.[25]

Nevertheless, Leśniewski's initial difficulties in using the new symbolic form of logical communication did not deter him from making further attempts. He had a positive model for such communication: "In the history of the establishment of mathematics, the most imposing embodiment of the achievements made since Greek times on the question of the soundness of the deductive method, is still for me the Grundgesetze der Arithmetik of Gottlob Frege. Frege's system is, however, an inconsistent system, as was demonstrated by Bertrand Russell, as is well known, when he constructed his celebrated 'antinomy', concerning 'the class of classes which are not their own elements.'"[26]

Between 1915 and 1918, while working in Moscow, Leśniewski established connections with various mathematicians, including Wacław Sierpiński.[27] During this period, Leśniewski favoured formal language as a simpler and more precise tool for expressing thoughts in the foundation of mathematics.

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[23] J. Jadacki: *Stanisław Leśniewski. Geniusz logiki*. Oficyna Wydawnicza Epigram, Bydgoszcz 2016, p. 22.

[24] S. Leśniewski: *On the Foundations of Mathematics…*, p. 182.

[25] Ibidem.

[26] Ibidem, p. 177.

[27] See J. Jadacki: *Stanisław Leśniewski...*, pp. 23–24.

As he recalled, this decision was also prompted by conversations with Leon Chwistek in 1920.[28] At the same time, he negated his earlier work: he declared "the bankruptcy of the 'philosophical'-grammatical work of the initial period of his work."[29]

The second formulation of Russell's antinomy (1927)

After reading Łukasiewicz's book and learning about Russell's contradiction construction, Leśniewski wrote: "problems concerning the antinomies were the most demanding subject of my deliberations for over eleven years."[30] This statement, from 1927, precedes by two years his highly formalistic achievement presented in *Grundzüge eines neuen Systems der Grundlagen der Mathematik*.[31]

One can say that in the first part of a series of articles *On the Foundations of Mathematics*, published in 1927, Leśniewski underestimated Russell's construction of antinomy and concentrated only on the understanding of class. He expected mathematics to maintain a 'connection with reality of any intuitive, scientific value.'[32] This is why he rejected the distributive understanding of class and instead developed a collective (mereological) interpretation. He began by describing the characteristics of the collective class.[33] One of the characteristics is that "no object is the class of classes not subordinated to themselves."[34] Russell's construction of antinomy is substantially weakened by neglecting the existence of an object, which is crucial to it. The situation can be illustrated as follows: "[…] seeing that no object can be a round square, I am not aware of any «antinomy» in the circumstance

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[28] S. Leśniewski: *On the Foundations of Mathematics*. [Chapter XI]…, pp. 364–382.

[29] S. Leśniewski: *On the Foundations of Mathematics*. [Introduction, Chapters I–III]…, p. 198. First publication: "O podstawach matematyki. Rozdział XI." *Przegląd Filozoficzny*, 34 (1931), pp. 153–170.

[30] Ibidem.

[31] S. Leśniewski: *Fundamentals of a New System of the Foundations of Mathematics*. In: Idem: *Collected Works*…, vol. 1, pp. 410–605. First publication: "Grundzüge eines neuen System der Grundlagen der Mathematik." *Fundamenta Mathematicae*, 14 (1929), pp. 1–81.

[32] S. Leśniewski: *On the Foundations of Mathematics*. [Introduction, Chapters I–III]…, p. 178.

[33] Ibidem, pp. 202–206.

[34] Ibidem, p. 204.

that leads to a contradiction, – from the assumption that a round square is round, together with the assumption that a round square is not round. I have ceased then to see the «antinomy» in Russell's construction, having ceased to believe in the class of classes which are not subordinated to themselves, thereby rejecting one of the fundamental steps in the above construction."[35] Furthermore, Leśniewski questioned Russell's reasoning. A collective class is not concerned with the following law: "if *K* is the class of objects *a*, and *P* is subordinated to class *K*, then *P* is *a* […]."[36]

The third formulation of Russell's antinomy (1938)

Leśniewski also worked on the problem of Russell's antinomy during the final years of his life. He was preparing a major publication on the antinomies. Unfortunately, his research was cut short by his untimely death in May 1939, and his notes were destroyed during the Warsaw Uprising in August 1944. Nine years later, based on his memory, Bolesław Sobociński published Leśniewski's final research on the possibility of constructing antinomies in Frege's revised system of logic from *Basic Laws of Arithmetic*. The result was devastating for Frege's logic, as it meant the system could not be improved. Sobociński's paper proved a significant flaw in Frege's proposed solution. According to Sobociński, Leśniewski justified it using his ontological model: if the domain satisfying the modified Basic Law V contains more than one object, Frege's theory becomes contradictory. Although one contradiction is eliminated, another emerges. This conclusion, therefore, highlights the failure of Frege's method of eliminating problems: it is difficult for such a radically monistic universe to be the subject of mathematics.

The archival document published below, written by Leśniewski in May 1938, sheds new light on his final research on the problem of the antinomy in Frege's logic.

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[35] Ibidem, p. 205.
[36] Ibidem, p. 206.

Later studies on *Frege's way out*

Vincent Frederick Rickey, Sobociński's doctoral student and one of the editors of the English edition of Leśniewski's work, considered Sobociński's publication to be a very important paper containing Leśniewski's third and definitive analysis of Russell's antinomy.[37] As one might think, this assessment is weakened by Rafał Urbaniak's criticism of certain formulas used by Sobociński, as well as the reasoning based on them. Urbaniak disagrees with Sobociński's ontological interpretation of 'being an element of the distributive class.'[38]

The objections to Sobociński's paper, which are outlined above, were expressed long after it was published. As one might assume, they are primarily relevant to those who study Leśniewski's views. Despite potential doubts regarding the accepted translation of the original formulas, the publication left a strong impression on scholars sympathetic to logicism and, consequently, interested in Frege's views as presented in *Basic Laws of Arithmetic*.

Willard Van Orman Quine and Peter Thomas Geach played a crucial role in popularising Leśniewski's reasoning by publishing papers on *Frege's way out*.[39] The article presents a generalized version of Leśniewski's reasoning, employing the language of *Principia Mathematica* but omitting Leśniewski's specific theoretical tools. The phrase *Frege's way out*, used in the article's title, rapidly became the standard term for Frege's method of eliminating Russell's antinomy, as outlined in the *Afterword* to his *Basic Laws of Arithmetic*. This subject has been extensively addressed in the literature.[40]

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[37] V. F. Rickey: "An Annotated Leśniewski Bibliography." In: S. Leśniewski: *Collected Works…*, vol. 2, p. 772.

[38] R. Urbaniak: "Leśniewski and Russell's Paradox. Some Problems." *History and Philosophy of Logic*, 29 (2008), pp. 133–134.

[39] W. V. O. Quine: "On Frege's Way Out." *Mind*, 64/254 (1955), pp. 145–159; P. T. Geach: "On Frege's Way Out." *Mind*, 259 (1956), pp. 408–409.

[40] M. Dummett: "Frege's Way Out: A Footnote to a Footnote." *Analysis*, 4/33 (1973), pp. 139–140; J. L. Hudson: "Frege's Way Out." *Philosophy Research Archives*, 1 (1975), pp. 135–140; G. Landini: "The Ins and Outs of Frege's Way Out." *Philosophia Mathematica*, 1/14 (2006), pp. 1–25; L. Linsky, G. F. Schumm: "Frege's Way Out: A Footnote." *Analysis*, 1/32 (1971), pp. 5–7; M. D. Resnik: "Some Observations Related to Frege's Way Out." *Logique et Analyse*, 7/27 (1964), pp. 138–144.

Conclusion

Stanisław Leśniewski dealt with the problem of Russell's antinomy for many years. He wrote the document published below in the third and final work period on the subject matter. Leśniewski's untimely death in May 1939 and the devastation of Warsaw during the Warsaw Uprising resulted in the absence of any publications by Leśniewski on the topic. The only surviving document is the two-page text published here, containing Leśniewski's draft proof of the sentence "a = b" in the reformed system of Frege's logic. This proof demonstrates that Frege's logic cannot be rendered free of antinomies as long as the universe contains more than one object. This issue, known as *Frege's way out*, has been extensively discussed in the literature. The document offers new insights into this debate. Leśniewski's original argumentation appears to differ from Bolesław Sobociński's reconstruction, which was based on his memory. We are conducting further in-depth research on this document.

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Das Grundschema eines Beweises für den Satz „a = b" im reformierten Fregeschen System.[1]

Zwei im folgenden vorausgesetzte bei Frege geltende Sätze über die „Wertverläufe"[2]

A1. $\dot{\varepsilon} f(\varepsilon) = \dot{\varepsilon} g(\varepsilon) \,.\, g(a) \,.\!\supset\!:\, a = \dot{\varepsilon} g(\varepsilon) \,.\!\vee\!.\, f(a)$

[vgl. Frege, Grundgesetze der Arithmetik, Band II, S.262, V'c].

A2. $[\mathfrak{a}] : f(\mathfrak{a}) \,.\!=\!.\, g(\mathfrak{a}) :\!\supset\!.\, \dot{\varepsilon} f(\varepsilon) = \dot{\varepsilon} g(\varepsilon)$

[vgl. b, c, V' und im Bande I S.69, Va][3]

T1. $\dot{\varepsilon} f(\varepsilon) = \dot{\varepsilon}(\varepsilon = a) .\!\supset\!:\, a = \dot{\varepsilon}(\varepsilon = a) \,.\!\vee\!.\, f(a)$ *[A1, $g(\xi)/\xi = a$]*

T2. $\dot{\varepsilon} f(\varepsilon) = \dot{\varepsilon} g(\varepsilon) \,.\, g(a) \,.\!\supset\!:\, a = \dot{\varepsilon} f(\varepsilon) \,.\!\vee\!.\, f(a)$

(1) $\dot{\varepsilon} f(\varepsilon) = \dot{\varepsilon} g(\varepsilon)$

(2) $g(a) .\!\supset\!:$

(3) $a = \dot{\varepsilon} g(\varepsilon) \,.\!\vee\!.\, f(a) :$ *[A1, 1, 2]*

$a = \dot{\varepsilon} f(\varepsilon) \,.\!\vee\!.\, f(a)$ *[3, 1]*

T3. $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big) = \dot{\varepsilon} g(\varepsilon) . g(a) \,.\!\supset\!.\, g\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big)$

(1) $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big) = \dot{\varepsilon} g(\varepsilon).$

(2) $g(a) .\!\supset\!.$

(3) $a = \dot{\varepsilon} g(\varepsilon).$ *[A1, $f(\xi)/\xi = \dot{\varepsilon} g(\varepsilon)$, 1, 2]*

(4) $a = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big).$ *[3, 1]*

$g\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big)$ *[4, 2]*

T4. $\dot{\varepsilon}(\varepsilon = a \,.\!\vee\!.\, \varepsilon = b) = \dot{\varepsilon}(\varepsilon = b \,.\!\vee\!.\, \varepsilon = a)$ *[A2, $f(\xi)/\xi = a \,.\!\vee\!.\, \xi = b$, $g(\xi)/\xi = b \,.\!\vee\!.\, \xi = a$]*

T5. $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = a \,.\!\vee\!.\, \varepsilon = b)\big) = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = b \,.\!\vee\!.\, \varepsilon = a)\big)$ *[T4]*

D [Definition]. $[\mathfrak{g}] : a = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}\mathfrak{g}(\varepsilon)\big) .\!\supset\!. \sim \big(\mathfrak{g}(a)\big) :=. * (a)$

[Der Beweis kőnnte – nur etwas műhsamer – auch ohne irgendwelche Definition durchgeführt werden][4].

[1] The basic schema of a proof of the proposition „$a = b$" in the reformed Fregean system.

[2] Two Fregean theorems about „value-ranges" that will be presupposed below.

[3] [comp. b, c, V' and Va, volume I, p. 69]

[4] The proof could also be – though somewhat more laboriously – carried out without any definition.

T6. $a = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big) . \supset : \sim \big(*(a)\big) . \vee . \sim \big(g(a)\big)$ *[D]*

T7. $\sim \Big(*\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} * (\varepsilon)\big)\Big)\Big)$ *[T6, $a/\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} * (\varepsilon)\big)$, $g(\xi)/*(\xi)$]*

T8. $\sim \Big(*\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big)\Big) . g(a) . \supset . g\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big)$

(1) $\sim \Big(*\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big)\Big).$

(2) $g(a) . \supset \therefore$

$[\exists \mathfrak{g}]:$

(3) $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big) = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} \mathfrak{g}(\varepsilon)\big).$ *[D, $a/\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)$]*

(4) $\mathfrak{g}\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big):$

(5) $\dot{\varepsilon} g(\varepsilon) = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big) . \vee . \dot{\varepsilon} g(\varepsilon) = \dot{\varepsilon} \mathfrak{g}(\varepsilon) \therefore$ *[T1, $a/\dot{\varepsilon} g(\varepsilon)$, $f(\xi)/\xi = \dot{\varepsilon}\mathfrak{g}(\varepsilon)$, 3]*

(6) $\dot{\varepsilon} g(\varepsilon) = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big) . \vee . g\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big):$ *[5,T2,$a/\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)$,$f(\xi)/g(\xi)$,$g(\xi)/\mathfrak{g}(\xi)$,4]*

$g\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big)$ *[6, T3, 2]*

T9. $\sim \big(*(a)\big)$ *[T8, $g(\xi)/*(\xi)$, T7]*

T10. $g(a) . \supset . g\Big(\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)\Big)$ *[T8, T9, $a/\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon} g(\varepsilon)\big)$]*

T11. $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = a . \vee . \varepsilon = b)\big) = a . \vee . \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = a . \vee . \varepsilon = b)\big) = b$ *[T10, $g(\xi)/\xi = a . \vee . \xi = b$]*

T12. $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = a)\big) = a$ *[T10, $g(\xi)/\xi = a$]*

T13. $\dot{\varepsilon}(\varepsilon = b) = a . \supset . \dot{\varepsilon}(\varepsilon = a) = b$

(1) $\dot{\varepsilon}(\varepsilon = b) = a . \supset :$

(2) $\dot{\varepsilon}(\varepsilon = b) = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = a)\big):$ *[1, T12]*

(3) $\dot{\varepsilon}(\varepsilon = a) = \dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = a)\big) . \vee . \dot{\varepsilon}(\varepsilon = a) = b:$ *[T1, $a/\dot{\varepsilon}(\varepsilon = a)$, $f(\xi)/\xi = b$, 2]*

(4) $b = \dot{\varepsilon}(\varepsilon = b) . \vee . b = \dot{\varepsilon}(\varepsilon = a)$ *[T1, a/b, $f(\xi)/\xi = \dot{\varepsilon}(\varepsilon = a)$, 2]*

$\dot{\varepsilon}(\varepsilon = a) = b$ *[3, 4, 2]*

T14. $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = a . \vee . \varepsilon = b)\big) = b . \supset : a = \dot{\varepsilon}(\varepsilon = b) . \vee . a = b$

(1) $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = a . \vee . \varepsilon = b)\big) = b . \supset :$

(2) $\dot{\varepsilon}(\varepsilon = b) = \dot{\varepsilon}(\varepsilon = a . \vee . \varepsilon = b):$ *[T13, a/b, $b/\dot{\varepsilon}(\varepsilon = a . \vee . \varepsilon = b)$, 1]*

$a = \dot{\varepsilon}(\varepsilon = b) . \vee . a = b$ *[T2, $f(\xi)/\xi = b$, $g(\xi)/\xi = a . \vee . \xi = b$, 2]*

T15. $\dot{\varepsilon}\big(\varepsilon = \dot{\varepsilon}(\varepsilon = b . \vee . \varepsilon = a)\big) = a . \vee . a = \dot{\varepsilon}(\varepsilon = b) . \vee . a = b$ *[T11, T5, T14]*

T16. $b = \dot{\varepsilon}(\varepsilon = a) . \vee . b = a$ *[T15, T14, a/b, b/a, T13]*

T17. $a = \dot{\varepsilon} * (\varepsilon)$ *[T16, $b/\dot{\varepsilon} * (\varepsilon)$, T2, $f(\xi)/*(\xi)$, $g(\xi)/\xi = a$, T9]*

T18. $a = b$ *[T17, T17, a/b]*

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