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Coincidence, Probability, Cognitive Error

Abstract: A lot has been written about coincidence, including research into the probabilistic, methodological, psychological and philosophical aspects of coincidence This paper presents some aspects of the role of coincidence in the human cognitive system, focusing in particular on probabilistic reasoning and logical and methodological errors in reasoning, associated with the perception and interpretation of coincidence. Due to the nature of things, it is not possible to omit the areas of research mentioned above.

Keywords: coincidence, probability, error, fallacy

1. What is coincidence?

1. It is not easy to say what coincidence is (see, for example, Diaconis, Moestller, 1989; Hand, 2014; Johansen, Osman, 2015). We mean by it an unusual occurrence of circumstances, a surprising and unlikely combination of two or more events that happen with no apparent connection with one another, which usually seems remarkable. It might be that the number of the train ticket one buys is exactly the same as the date of one's birth. Or one learns a new foreign or mother tongue word and on the same day hears the same word in a film or reads it in a newspaper. The accounts of such events are often anecdotal and because of their peculiarity are worth relating: for example a clock in the house of an ill person stops at exactly the moment of his or her death, or in the same student group there are people with surnames "Gun" and "Bomb," or the driver into whose car one bumped at

a supermarket parking turns out one week later to be the boss in one's new work. The Internet has many examples of fascinating coincidences associated with famous people or events. One such popular coincidence is related to John Paul II and the unusual accumulation of "thirteens." In his Polish name "Jan Paweł Drugi", there are exactly 13 letters. When Karol Wojtyła was elected Pope, he was 58 years old, and 5 + 8 = 13; he died at the age of 85 (8 + 5 = 13), on the second of April 2005 (2 + 4 + 2 + 0 + 0 + 5 = 13) at 21 : 37 (2 + 1 + 3 + 7 = 13), in the 13th week of the year. His pontificate lasted 9031 days (9 + 0 + 3 + 1 = 13), 26 years and 5 months (2 + 6 + 5 = 13). These are but some of the thirteens associated with the Polish Pope. A lot more can be found in the Internet.[1] Even though some thirteens are evidently far-fetched, their number is impressive.

2. Coincidence always involves co-occurrence of two or more events (features, relationships) A, B, C… that happen simultaneously or in a specific order within the framework of a single situation that brings them together. The characteristic feature of coincidence is that it is surprising, astonishing or even sensational (Falk, Macgregor, 1983). Coincidences are perceived first of all as unlikely events. Indeed, the probability that eight random digits in a train ticket number will combine to one's date of birth (in the format: yyyy-mm-dd) is 0.00000001. Similarly, given that out of 38 million Polish citizens about 1,000 have the surname "Gun" and another 3,200 — the surname "Bomb," the probability that in a random group of 50 persons there are people with the two respective surnames is very low, approx. 0.00001. However, is it right to say that coincidences are just unlikely events and because of that they are astonishing? A deeper analysis shows that the relationship between low probability and astonishment is more complex than that. As was convincingly shown by Teigen and Keren (2003) and Griffiths and Tenenbaum (2007), events, even if they are very unlikely, do not usually astonish us. The probability that an eight-digit train ticket number will be equal to, let us say, 35106042, is the same as the probability that it will coincide with one's date of birth, i.e. it is 0.00000001. Any set of 8 random digits has the same probability of occurrence. Astonishment comes only if a combination of digits represents a certain value, an unusual system or meaning (see 4 below). The probability that in a group of 50 people there will be one with

[1] http://stepa.w.interiowo.pl/papiez1.htm (visited on 15.02.2016) see also http://www.wiadomosci24.pl/artykul/numerologia_jana_pawla_ii_189303.html (acces 15.02.2016).

the surname “Radkowski” and another with the surname “Witecki” is many times lower than the combination of the names “Gun” and “Bomb”;[2] however, the first is not surprising. Astonishment (Teigen, Keren, 2003) results rather from the contrast between what we expect and what really happens. It would be unusual to see that numbers drawn in a Lotto game[3] are 14, 15, 16, 17, 18, 19, because we are accustomed to the fact that Lotto draws do not combine into any evident system, and consequently we expect a sequence that does not follow any intrusive pattern. Many people think that the reason for astonishment is apparent lower probability of a sequence of subsequent numbers than “normal” numbers. Obviously, the terms “being accustomed” and”expectations” are subjective. Certain results may be surprising for some but not for others. For example, most people will not think the sequence of numbers 3, 5, 8, 13, 21, 34 unusual, but a mathematician will know that they are subsequent numbers in the Fibonacci series,[4] and it is easy to verify that drawing Fibonacci series numbers in a Lotto game is much less probable than drawing a sequence of subsequent numbers. Another example of the discrepancy between astonishment and probability: a grandfather handed out 20 candies to his 5 grandchildren, at random. One would think it unusual if every grandchild received exactly 4 candies, i.e. if the result was 4, 4, 4, 4, 4, rather than 3, 5, 4, 2, 6, which seems “typical.” On the other hand, the former result is much more probable than the latter (although it should be noted that it, too, is very unlikely, the probability being 0.0032). To conclude, astonishment is not a good measure of probability. It may be that events that are not in the least surprising are less probable than those that cause astonishment, as is shown by the above examples. It should be noted that an important feature of coincidence is a combination of two or more specific features that unite respective situations. An event may be surprising and astonishing because of its low probability, even if it is not a coincidence. An example would be unexpected arrival of a kangaroo at one’s apartment, which definitely is improbable, but has no traits of coincidence, as it does not have the abovementioned set of analogous features.

[2] In Poland, there live 560 people with the surname Radkowski and 790 with the surname Witecki.

[3] This is about a Lotto game where you draw 6 out of 49 numbers, meaning that there are 14 million possible combinations.

[4] A sequence of numbers where the subsequent number, starting from the 3rd, is the sum of the two previous numbers. The first two numbers are 0, 1, followed by 1, 2, 3, 5, 8…

3. Other important features of coincidence are its personal dimension and association with an individual person. The fact that a ticket number corresponds to a certain date is not surprising in itself, unless the date is important for someone. Research (Falk, 1989) shows that people are more surprised by coincidences that concern them personally than by those that happen to others, even if these are the same or comparable events.

4. Researchers agree that probably the most important feature of coincidence is its meaningfulness (see Diaconis, Moestller, 1989; Nickerson, 2004, p. 51), despite the lack of a cause and effect relationship between events. The term "meaningfulness" is hard to define. Obviously, "meaning" may relate to the interpretation of the elements of a situation as broadly understood signs forming semantic compounds whose meaning comprises — also indirectly, e.g. via associations — other elements of the situation. In this sense, a ticket number coinciding with one's date of birth is "meaningful." Similar is the case when a clock stops at exactly the moment of one's death: the stopping of a clock may be seen as a symbol of the end of life. However, researchers describe as "meaningful" events that evidently do not fit into the above system. For example, Hand (2014, p. 43) presents as meaningful an event that happened on September 11, 2011. One of the American military agencies intended to perform on the morning of that day a simulation of an accident, where, as a result of failure, an airplane crashes into a building. At that very time, an airplane seized by terrorists hit the Pentagon building. Those events belonged to the same category, although it is doubtful whether there was any semantic relationship between them. There is no such relationship in a situation that is often given as an example of coincidence, where two people call each other at exactly the same moment, although they did not arrange that and there is no evident reason for that.

Undoubtedly, many coincidences may be seen as meaningful also in the sense that they seem to evoke a message, like the thunderbolt that struck St. Peter's Basilica on the day when Pope Benedict XVI resigned from office (February 11, 2013). Coincidences are generally regarded as a proof of the existence and activity of beings from the beyond that control the course of events and try to enigmatically signal something or send a mysterious sign. "This just cannot be a random chance" — is what one often thinks when observing a coincidence. According to psychological research, coincidences support belief in paranormal phenomena, such as telepathy, second sight or prophetic dreams. It seems that such beliefs are strongly associated with the tendency to make various cognitive errors, including errors in understanding probability and using the term (Brugger et al., 1995; Bressan, 2002; Musch, Ehrenberg, 2002).

5. Significant coincidence was the topic of Carl S. Jung's research (Jung, 1969) in association with his concept of "synchronicity." He defined meaningful as "informative, emotionally charged, and transforming the observer's beliefs or point of view." This means that the meaningfulness of coincidence is in fact about influencing the person experiencing coincidence. As an example, he described an event that happened at an important point of the therapy of one of his female patients (Jung, 1969, pp. 109—110).

> She had an impressive dream the night before, in which someone had given her a golden scarab — a costly piece of jewellery. While she was still telling me this dream, I heard something behind me gently tapping on the window. I turned round and saw that it was a fairly large flying insect that was knocking against the window-pane from outside in the obvious effort to get into the dark room. This seemed to me very strange. I opened the window immediately and caught the insect in the air as it flew in. It was a scarabaeid beetle, or common rose-chafer (Cetonia aurata), whose gold-green colour most nearly resembles that of a golden scarab.

Jung claimed that the phenomenon he called "synchronicity," by which he understood acausal significant coincidences, could not be explained by the laws or statistics or the laws of science based on the principle of casuality. Unfortunately, Jung's research is limited to discussing and categorising various cases of coincidence from the perspective of his analytical psychology. The empirical data he uses are too unclear and doubtful to confirm his theses, which, by the way, seem to be formulated in a way that prevents any empirically verifiable conclusions.

2. Probabilistic aspects of coincidence

1. An analysis of the phenomenon of coincidence must take into account the reasons why certain events are considered to be unlikely.

Both objective causes of the occurrence of improbable events and subjective factors making people perceive certain events as unusual coincidences are important.

2. Events the probability of which is more than 0 but less than 1 are unforeseeable, even though, from the point of view of the quality of decision-making, there is a difference between events whose probability is close to 1 and those whose probability is close to 0. The former should be treated as if they were certain and the latter — as if they were not about to occur. When you think about it, it seems that this recommendation works in practice. What we regard as quite certain is in fact only highly probable and we make no distinction between "impossible" and unlikely. Émile Borel, the famous French mathematician, formulated the Single Law of Chance reflecting this intuition: "Phenomena with very small probabilities do not occur" (Borel, 1962, p. 1). However, other researchers suggest that the world is full of unlikely events (see Hand, 2014, pp. 8—12; Littlewood, 1953, pp. 104—107; Dembski, 1998, pp. 2—7). Moreover, it seems inevitable that unlikely events will happen. To understand the nature of the resulting paradox, it is important to think how the Borel's principle should be understood so that it corresponds to other probabilistic regularities. First of all, it should be applied only to individual random events that could in the future lead to an improbable result. After all, if you look at things that have already happened, their probability was usually very low in the past. If we throw a coin 100 times and note down whether it is heads or tails each time, then we will obtain a sequence like, for example, HHTHTTTH… Every such sequence has identical, cosmically low probability of $8 \cdot 10^{-31}$.[5] If we choose any of the possible sequences before actually throwing the coin, we may be certain that it will not arrive — this is where the Borel's law applies. However, at the same time, we know that *one* of the sequences *must* arrive. Bearing this phenomenon in mind, D.J. Hand formulates "The Law of Inevitability," which simply says that *something must happen* (Hand, 2014, p. 75). Some future events have many possible options that happen in an unforeseeable way and each of them is very unlikely, but logically, one of these options must happen. An interesting illustration was proposed by Stanisław Lem in his book *A Perfect Vacuum* (Lem, 1998, pp. 158—159). In this book, Professor Benedykt Kouska (a Czech) comes to the conclusion that the probability of his (Professor Kouska's) coming into the world

[5] By comparison, the probability of guessing 6 numbers in a Lotto game is $7 \cdot 10^{-8}$.

was a miracle of miracles and an event that was almost absolutely improbable:

> […] A certain army doctor, during First World War, ejected a nurse from the operating room, for he was in the midst of surgery when she entered by mistake. Had the nurse been better acquainted with the hospital, she would not have mistaken the door to the operating room for the door to the first-aid station, and had she not entered the operating room, the surgeon would not have ejected her; had he not ejected her, his superior, the regiment doctor, would not have brought to his attention his unseemly behaviour regarding the lady (for she was a volunteer nurse, a society miss), and had the superior not brought this to his attention, the young surgeon would not have considered it his duty to go and apologise to the nurse, would not have taken her to the café, fallen in love with her, and married her, whereby Professor Benedykt Kouska would not have come into the world as the child of this same married couple.[6]

The nurse mistaking the door was but one in a series of many events: on that day, the young surgeon substituted for his colleague who had had an accident, which had also been caused by a series of random events. A thorough calculation of the chances of Kouska's birth may also take into account the broader context including the fact that Archduke Ferdinand was shot in Sarajevo

> … for had he not been shot, war would not have broken out, and had war not broken out, the young lady would not have become a nurse; moreover, since she came from Olomouc and the surgeon was from Moravska Ostrava, they most likely would never have met, neither in a hospital nor anywhere else. One therefore has to take into account the general theory of the ballistics of shooting at archdukes, and since the hitting of the Archduke was conditioned by the motion of his automobile, the theory of the kinematics of automobile models of the year 1914 should also be considered, as well as the psychology of assassins, because not everyone in the place of that Serb would have shot at the Archduke, and even if someone had, he would not have been on target — not if his hands were shaking with excitement; therefore the fact that the Serb had a steady hand and eye also has its place in the probability distribution of the birth of Professor Kouska.[7]

[6] Translated from Polish by Michael Kandel.
[7] Translated from Polish by Michael Kandel.

Let me comment on the above example in the following way: Even if on the day of the outbreak of First World War the probability of the coming into the world in the next few years of a man of a genotype identical with that of Professor Kouska was very low, some people with some genotypes were indeed bound to come into the world, and each time their respective genotypes were — from the perspective of the day of the outbreak of the war — exceptionally improbable.

3. Another limitation to Borel's principle is the fact that repeating the same experiment many times increases the probability of the appearance of any event, even the most improbable (provided its probability is non-zero). The Law of Truly Large Numbers formulated by Diaconis and Moestler (1989, p. 859) says that "with a large enough sample, any outrageous thing is likely to happen." The name of the law is a reference to The Law of Large Numbers, known from the probability theory, which in its simplest form defined by Jacob Bernoulli says that if you repeat many times the same experiment whose result may be X with the probability of p, the frequency of the result X will approach p (cf. Feller, 1968, pp. 152—153). Thus, if you throw a coin many times, where the probability of throwing heads is $p = 0.5$, and if you calculate the frequency of heads, i.e. if you divide the number of heads by the number of throws, the result is number f that is close to 0.5, and the more times you throw the coin, the closer to p will the number f get. On the other hand, The Law of Truly Large Numbers claims something different: if you repeat an experiment a sufficient number of times, the least probable result will appear at least once. For example, if the probability that the result of an experiment will be X is one in a million, then if you repeat that experiment 4.6 million times, then it is hugely probable (0.99) that you will obtain the result X at least once.[8] This law works for number lotteries. The probability that you hit all 6 numbers in a Lotto is one in 14 million, but so many bets are filled in that almost always someone gets the right numbers. This law should be applied to explain what is called "prophetic dreams." Sometimes people dream things that actually happen to them some time later. The coincidence between the dream and reality may at times be striking. One of the most famous historically recorded stories of this kind concerns Abraham Lincoln, the US President assassinated in 1863. A few weeks before his death, Lincoln

[8] Generally speaking, if the result X of an experiment has the probability of $1/n$, then by repeating the experiment $4.6 \cdot n$ times, the probability that X will appear at least once is around 99%.

had a dream of himself in the White House crowded with sobbing people. When he entered (in his dream) the East Room, he saw body dressed in funeral clothes and lying on a catafalque. Someone told him that the US President had been assassinated. The dream kept Lincoln in shock for many days (Lamon, 1994, pp. 116—117). Interestingly, after his death, Lincoln's body was indeed exhibited to the public in the East Room of the White House and, as the chroniclers report, was visited by crowds of people deeply moved by the President's death. The Law of Truly Large Numbers may be used to try and estimate the probability of "prophetic dreams." Let us imagine that a list has been made for every individual person with descriptions, each consisting of several sentences, that are selected in such a way that no matter what happens tomorrow in the life of a given individual, this can be fitted to one of the items on the list. The list contains cases like "unexpected inheritance from an uncle in America," "hospitalization after being bitten by a dog," "finding a corpse in one's garden" or "a row at a wedding." The descriptions are each time tailored to the respective individual: his or her sex, place of residence, and family status — if one does not own a garden then, obviously, one cannot possibly find a corpse there. No matter what an individual dreams about him or herself, it is there on the list — which is exhaustive. Accordingly, an individual's dream (about him or herself) is a shot that has one in a million chances of being a hit. Let us then assume that every night only one in a hundred persons dreams about him or herself. Given the number of inhabitants in Europe, every day, 5 million dreaming Europeans "give a shot." Accordingly (see footnote 6), almost certainly one European per day experiences an unusual, prophetic dream that becomes reality the next day. Perhaps once a month an individual has an exceptionally accurate dream with a number of striking details. Similar calculations may be done with much weaker assumptions, for example assuming that the "list of dreams" includes 10 million entries and that only one in a thousand persons has a dream about him or herself every night. The result will always be the same: given the billions of dreaming people over the years, some dreams must come true.

4. Another factor that makes people regard certain phenomena as unusual coincidences is *selection*, i.e. ignoring the fact that a certain phenomenon is the effect of numerous repetitions of the same experiment. When the bulk of results is omitted and only one particular result is selected, it may seem to you that you witness something unusual. According to Falk (1981—1982, p. 24): "One characteristic of coincidences is that we do not set out to seek them in a predeter-

mined time and place; they simply *happen* to us. Since they stand out in some strange combination, we single them out and observe them under a magnifying glass." A good illustration of this mechanism are "miraculous recoveries." Most diseases, even the gravest ones, sometimes remit without any concrete reasons. If we take a specific case of such unexpected recovery under a magnifying glass, it may turn out that it happened in special circumstances, e.g. that the ill person used an unusual therapy (e.g. bat's fat or homeopathy) or visited "miraculous" places of religious cult. C. Sagan (Sagan, 1997, pp. 220—221) analyzed the cases of recovery from cancer among people who had visited Lourdes, France, where there is a spring of holy water. He discovered that the percentage of cured pilgrims is less than the percentage of patients who experienced spontaneous remission. Thus, it seems that the belief in the curing effect of the holy water is wrong. Perhaps places like Lourdes are famous because of the publicity of selected cases of recovery. The millions whose condition did not improve are disregarded.

Here is another example of selection error, this time concerning statistical tests. Statistical verification of the hypothesis that a population includes 10% of elements that have a feature C is done by means of a statistical test based on the assumption that if the hypothesis is correct, i.e. the population in question includes 10% of elements that have the feature C, then a random sample of that population of, for example, 1,000 elements should include, with the probability of 0.95, from 81 to 120 elements that have the feature C. Accordingly, if the sample of 1,000 elements includes more or fewer elements with the feature C, then the hypothesis that the population contains 10% elements with the feature C is dismissed. It is then concluded that the hypothesis was dismissed at the significance level of 0.05. It should be noted that if a population includes exactly 10% of elements with the feature C, then with the probability of 0.05, i.e. one in 20 cases, the number of elements with the feature C will fall outside the 81—120 span. If this is the case, the decision to dismiss the 10% hypothesis is of course wrong. Let us now imagine that a population indeed includes 10% of elements with the feature C and that 20 researchers perform the above verification independently, by taking a random sample of 1,000 elements and checking the number of elements with the feature C. It should be expected that one researcher will, by mere chance, obtain a false result: he or she will dismiss a true hypothesis. Similar is the case with all statistical tests. There is always a predefined probability α that a hypothesis will be dismissed even though it is true. The value of α is called the level of significance of the test, and it is usually $\alpha = 0{,}05$. If the same test is independently repeated more times, the probability that at least in one case the result will dismiss the hypothesis that is being verified grows. Let us imagine that a pharmaceutical company wants to prove the effectiveness of a drug that in fact is ineffective. The company has that drug tested by 20 laboratories. It is very probable that one of the laboratories obtains, by chance, a statistically significant result that dismisses the hypothesis that the "drug does not work." The company conceals the fact that the other 19 laboratories arrived at a different result and publishes information that "the effectiveness of the drug has been confirmed by research."

A similar effect should be expected when the same sample is analyzed for more features. It may be easily calculated that, at the level of significance 0.05, when analyzing k true hypotheses concerning independent features, at least one such true hypothesis will be dismissed with the probability of $1 - 0.95^k$. In epidemiology, this error is called *multiple comparisons fallacy*. An example could be the research conducted in Sweden in the 1990s on the health effects of living near electric power lines, which led to the conclusion that children who live near such lines suffer from leukemia more often than the average population.[9] However, the result was wrong as it was obtained on the basis of a sample of people living near electric power lines analysed for 800 different ailments! A total of 800 hypotheses were made, each of which concerned some kind of illness and assumed that the rate of people suffering from that disease in the examined population was the same as in the entire Swedish population. Even if the level of significance of that research was 0.001, the probability that one of the hypotheses would be dismissed by chance is more than 0.5.

3. Coincidences and probabilistic reasoning

1. Coincidence is associated with a very important type of probabilistic reasoning, where the occurrence of an improbable event is

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[9] http://www.pbs.org/wgbh/pages/frontline/programs/transcripts/1319.html (visited on 02.04.2016).

treated as evidence of a certain thesis. If the police finds out that the same individual was in three different locations of a bomb attack, then they will certainly find reasonable the hypothesis that the individual may be linked with the attacks. It is too unlikely that this was by chance — such would be the justification of their supposition. The problem in this context is: was it a mere coincidence or is there another explanation, usually causal? If you walked down empty streets and noticed a stranger walking behind you in the same direction, you might be surprised if the stranger turned into the same streets as you. After the first or second turn, you would think: "it is only a coincidence." However, after the fourth or fifth turn, you might conclude that the stranger's behaviour needed special explanation and was hard to attribute to "mere coincidence," because the probability was too low. Similarly, if only 20 students were admitted to a prestigious university course and 10 out of them were friends and relatives of the dean, the question would inevitably appear: "Was it a coincidence or nepotism?" Coincidence is a very typical stimulant for conspiracy theories. The reasoning is the same as in the above examples: "it cannot have been a coincidence." For example, in a live account of the 9/11 terrorist attack, the BBC TV channel reported that building 7 had collapsed while it was still standing. It collapsed only 20 minutes later.[10] A coincidence? Such misinformation might be explained by a simple, though unusual, mistake caused by the tension accompanying the sudden and tragic events. On the other hand, it could be explained as evidence that the 9/11 events — as the conspiracy theory has it — were staged with the BBC clumsily playing the role the conspirators wanted it to play.

2. The very low probability of self-creation of the first living cell, which started the evolution of life on the Earth (Mullan, 2002), is the basis to claim that life was created by some intelligent being (intelligent design). This kind of reasoning corresponds to the scheme presented in detail below.

3. The reasoning in question takes into consideration and compares two hypotheses: Hypothesis H_0 "random chance" and hypothesis H_1 "cause X." In the light of the "random chance" hypothesis, an event has low probability, whereas "cause X" makes the same event much more probable. Let us start a detailed analysis of the process of comparing the hypotheses with a simple, albeit somewhat artificial, example. Imagine that you ask two persons to independently write down on covered pieces of paper a total number higher than

[10] https://www.youtube.com/watch?v=677i43QfYpQ (visited on 02.04.2016).

zero and not more than 1,000. If, having checked the two pieces of paper, it turns out that the two persons wrote the same number 871, the thought will certainly cross your mind that although you asked them to choose numbers independently, they either consulted each other or one cribbed the number from the other. What is the reason for such conclusion? The answer is quite obvious: it is too unlikely to be a coincidence. Indeed, the probability that both the first and the second person randomly write down number 871 is one in a million, so it is very low. However, the probability that the first person writes down 327 and the second person writes down 786 is one in a million, too; and still, if this were the case, there would be no reason to conclude that they did not act individually. This is because the conclusion is based not on the fact that both the first and the second person chose number 871, but on the fact that they chose the same number. The probability of drawing twice the same number from a sequence of numbers from 1 to 1,000 is one in a thousand, so it is also very low. So what is the course of this reasoning? What are its motivations and stages? This reasoning involves comparing two hypotheses that explain an event e, where two persons write down identical numbers. According to H_0, the selection of each of the two persons was random and independent from the other person. On the other hand, according to H_1, the two persons consulted each other. Bayes method compares two numbers: $P(e|H_0)$, i.e. the probability of event e assuming H_1 and $P(e|H_1)$ — the probability of event e assuming H_1. The value $P(e|H)$ is the *likelihood* of hypothesis H given evidence e. It can be regarded as the extent to which hypothesis H explains e. By calculating the likelihood ratio of hypotheses H_0 and H_1 given e:

(A) $$\beta = \frac{P(e|H_0)}{P(e|H_1)}$$

you find out which of the two hypotheses is favoured by e and to what extent. If $\beta = 1$, the two hypotheses have identical value (e is irrelevant and has no evidential power). If $\beta > 1$, e favours H_0 the more the higher is β. If $\beta < 1$, e favours H_1 the more the closer to 0 is β. In the above case, $P(e|H_0) = 0.001$ and $P(e|H_1) = 1$, so $\beta = 0.001$. However, this calculation does not exhaust the analytical process. So far, we have found out that the likelihood of H_0 is 1,000 times lower than the likelihood of H_1. Yet, we are interested in the probability ratio of the two hypotheses, i.e. the ratio of $P(H_0|e)$ to $P(H_1|e)$. Before presenting the formal solution to the problem, it should be noted that the assessment of the probability that the two persons consulted each other depends on a factor that has not yet been mentioned. This assessment would be completely different if the two persons were doing the task sitting at the same desk and not being controlled in any way than if they were in two separate rooms or did not even know about each other's existence. In the second case, despite very low probability of the two numbers being the same, the conclusion would be that it was *a mere coincidence*. The hypothesis of consultation would be raised only if it was realistic.

4. The above circumstances are provided for in the following Bayes formula:

$$\text{(B)} \qquad \frac{\mathrm{P}(H_0|e)}{\mathrm{P}(H_1|e)} = \frac{\mathrm{P}(e|H_0)}{\mathrm{P}(e|H_1)} \times \frac{\mathrm{P}(H_0)}{\mathrm{P}(H_1)}$$

that enables us to compare the two probabilities concerned (on the left). Let us rewrite the formula using the above symbols (A) as

$$\frac{\mathrm{P}(H_0|e)}{\mathrm{P}(H_1|e)} = \beta \times \frac{\mathrm{P}(H_0)}{\mathrm{P}(H_1)}$$

It can be seen that the analysis must take into account the quotient of a priori probabilities of both hypotheses,

$$\text{(C)} \qquad \frac{\mathrm{P}(H_0)}{\mathrm{P}(H_1)}$$

which should be multiplied by β. The result represents the proportion of likelihoods on the left side of the equation (A). Assuming that H_0 and H_1 are the only hypotheses that count, meaning that $\mathrm{P}(H_0|e) + \mathrm{P}(H_1|e) = 1$, it is possible to calculate their respective probabilities:

$$\mathrm{P}(H_0|e) = \frac{\mathrm{P}(e|H_0)\mathrm{P}(H_0)}{\mathrm{P}(e|H_0)\mathrm{P}(H_0) + \mathrm{P}(e|H_1)\mathrm{P}(H_1)}$$

whereas $\mathrm{P}(H_1) = 1 - \mathrm{P}(H_0)$.

5. The two elements of the above analysis are used by Griffiths and Tenenbaum (2007) to distinguish between mere coincidence and a rational argument for a certain hypothesis. Their model always contains the "coincidence" hypothesis and the "causal" hypothesis. You calculate likelihood given e for both hypotheses and analyse their

ratio, which was above denoted as β. If β is very low, then e speaks strongly in favour of the causal hypothesis. Then you probably deal with something more than mere coincidence. However, in order to establish high likelihood of the causal hypothesis, β should be multiplied by (C), i.e. the ratio of *a priori* probability of the H_0 and H_1 hypotheses, respectively. If the product is much below 1, the verdict is: mere coincidence. The H_1 hypothesis is assumed, if the product is close to 1.

6. It should be noted that the above scheme gives precise results only if we know the “input” parameters. Unfortunately, in most cases these values may only be roughly calculated and sometimes it is not even certain if speaking of their likelihood is at all relevant. For example, considering the abovementioned case of the unusual information broadcast by the BBC (e), there are two possible hypotheses: H_0 — “coincidence” and H_1 — “conspiracy.” The values $P(e|H_0)$ and $P(e|H_1)$ should be inserted in the formula, but they cannot possibly be determined in a reliable way. How do you calculate the probability $P(e|H_1)$ that BBC would broadcast that information given that BBC was involved in the conspiracy? It can only be concluded that value β is low, but *a priori* probability that the BBC was involved in a conspiracy that simulated the WTC terrorist attack is very low, which makes the second part of the equation (B) very high. Thus, the conspiracy theory is poorly substantiated.

Despite problems with practical application of the above model, it does have some advantages. The model shows a valuable direction for analyzing coincidence. One of the most typical errors is analyzing only β and disregarding the *a priori* likelihood of the hypotheses in question.

7. A very important and frequent error results from — using the categories of the above model — understating the $P(e|H_0)$ value. It is not taken into account that e should be treated as the effect of a coincidence rather than a specific cause. This error is discussed at length in Taleb (2001) — people very often find a causal connection between different events, not aware of their actual randomness. For example, numerous data suggest that the financial success of many stock market players is purely coincidental, even though they are under the illusion that they have worked for their success.

Conclusion

Coincidences often cause irrational human reactions. They may be wrongly interpreted, often emotionally and on the basis of erroneous evaluation leading to unjustified conclusions. However, as is proved above, it is possible to apply statistical and probabilistic methods to analyse coincidences. It turns out that unlikely events may trigger fully justified conclusions concerning causal dependencies. Moreover, it is possible to identify the reasons behind typical human errors. Obviously, this article does not exhaust the topic and is limited to presenting only some of its aspects.

Bibliography

Borel É., 1962: *Probability and Life*. Dover Publications.

Bressan P., 2002: *The connection between random sequences, everyday coincidences, and belief in the paranormal*. "Applied Cognitive Psychology" 16, pp. 17—34.

Brugger P., Regard M.A., Landis T., Graves R.E., 1995: *The roots of meaningful coincidence*. "Lancet" 345, pp. 1306—1307.

Dembski W., 1998: *The Design Inference: Eliminating Chance through Small Probabilities*. Cambridge, Cambridge University Press.

Diaconis P., Moestller F., 1989: *Methods for Studying Coincidences*. "Journal of the American Statistical Association," Dec., Vol. 84, No 408, pp. 853—861.

Duane G.S., 2015: *Synchronicity from Synchronized Chaos*. "Entropy" 17, pp. 1701—1733.

Falk R., 1981—1982: *On coincidences*. "Skeptical Inquirer" 6, pp. 18—31.

Falk R., 1989: *Judgment of Coincidences: Mine versus Yours*. "The American Journal of Psychology" 12(1), Vol. 102, Issue 4, pp. 477—493.

Falk R., Macgregor D., 1983: *The surprisingness of coincidences*. In P. Humphreys, O. Svenson, & A. Vari (Eds.), *Analysing and Aiding Decision Processes*. Budapest, Adademiai Kiad6, pp. 489—502.

Feller W., 1968: *An Introduction to Probability Theory and Its Applications*. Vol. 1. New York, John Wiley and Sons, Inc.

Griffiths T.L., Tenenbaum J.B., 2007: *From mere coincidences to meaningful discoveries*. "Cognition" 103, pp. 180—226.

Hand D.J., 2014: T*he Improbability Principle. Why Coincidences, Miracles, and Rare Events Happen Every Day*. New York, Scientific American, Farrar, Straus and Giroux.

Johansen M.K., Osman M., 2015: *Coincidences: A fundamental consequence of rational cognition*. "New Ideas in Psychology" 39 (October), pp. 34—44.

Jung C.G., 1969: *Synchronicity: An Acausal Connecting Principle*. Princeton, Princeton University Press.

Lamon W.H., 1994: *Recollections of Abraham Lincoln 1847—1865*. Lincoln, University of Nebraska Press.

Lem S., 1998: *Apokryfy*. Kraków, Wydawnictwo ZNAK, pp. 158—159 [in Polish].

Littlewood J.E., 1953: *A Mathematician's Miscellany*. London, Methuen.

Mullan D.J., 2002: *Probabilities of randomly assembling a primitive cell on Earth*. "PCID" vol. 1.4., (October-December), p. 4.

Musch J., Ehrenberg K., 2002: *Probability misjudgment, cognitive ability, and belief in the paranormal*. "British Journal of Psychology" 93, pp. 169—177.

Nickerson R.S., 2004: *Cognition and Chance: The Psychology of Probabilistic Reasoning*. Mahwah, Jersey, Lawrence Erlbaum Associates, pp. 48—70.

Sagan C., 1997: *The Demon-Haunted World. Science as a Candle in the Dark*. London, Headline Book Publishing.

Taleb N.N., 2001: *Fooled by Randomness: The Hidden Role of Chance in Life And in the Markets*. New York, Random House.

Teigen K.H., Keren G., 2003: *Surprises: low probabilities or high contrast*? "Cognition" 87, pp. 55—71.