Marek Otisk
Filosofický ústav Akademie věd ČR, Praha; Filozofická fakulta, Ostravská univerzita

Descriptions and Images of the Early Medieval Latin Abacus

I

The position of arithmetic in early Medieval Latin schools was in many ways exceptional. This art was regarded as the first of the *quadrivium*[1] and its role was comparable with the role of logic (dialectics) within the *trivium* arts[2]. Among the authors of the most authoritative and important texts were Aurelius Augustinus, Martianus Capella, Isidorius of Seville[3], Cassiodorus, and Boethius. Their texts served as an argument supporting the growing interest in this art as well as functioned as teaching material.

References to the abacus, a mathematic tool used for basic arithmetic calculations (addition, subtraction, halving, multiplication, doubling, and division), can be found in the texts as early as those written between the 4th and

---

[1] A.M.S. Boethius: *De institutione arithmetica* I, 1. Ed. H. Oosthout; J. Schilling: *Corpus Christianorum, Series Latina* 94A. Turnhout 1999, pp. 12—13, or: F.M.A. Cassiodorus: *Institutiones divinarum et humanarum litterarum* II, 4, 1. Ed. R.A.B. Mynors. Oxford 1961, p. 132.

[2] Comp. R. Franci: *L'insegnamento dell'aritmetica nel Medioevo*. In: *Itinera mathematica. Studi in onore di Gino Arrighi per il suo 90° compleanno*. Eds. R. Franci, P. Pagli, L.T. Rigatelli. Sienna 1996, pp. 12—13.

[3] See Augustinus Hipponensis: *De libero arbitrio* II, 3—13. Ed. W.M. Green. *Corpus Christianorum, Series Latina* 29. Turnhout 1970; Martianus Capella: *De nuptiis Philologiae et Mercurii*. Ed. J. Willis. Leipzig 1983; *Isidori Hispalensis episcopi Etymologiarium sive Originum libri XX*. Ed. W.M. Lindsay. Oxford 1911.

7th century[4]. However, during the period between the 7th and 9th century, we have no evidence of active use of any form of this tool in the Latin Christian environment[5]. Nevertheless, a significant change took place in the last quarter of the 10th century, when monastic and cathedral schools gradually began using the abacus, as can be observed in preserved written sources referring to abacus calculations[6]; it was also in this period that the first texts dedicated to methods of calculation on the board abacus were written, accompanied by images depicting this calculating table.

It is precisely the abacus of the Latin pre-scholastic Middle Ages which will be the main object of interest of this article. By analysing the two oldest surviving descriptions of the abacus (by Richer of Reims and Bernelius the younger from Paris), together with a pair of the oldest images of the calculating tool preserved in manuscripts (between the end of the 10th and beginning of the 12th century), I will attempt to provide a detailed description of this arithmetic tool which proved to be a source of great fascination in its time for many intellectuals, and which is also known as the Gerbert's abacus (after the civil name of Pope Silvester II) in period texts. Due to its frequent use in monastic schools, it is also often called the monastic abacus.

After providing a brief historical overview, the focus will shift mainly to the description and analysis of the individual parts of the abacus, while primary emphasis will be put on the surviving descriptions and images of the tool from the end of the 10th and beginning of the 11th century as well as mutual comparisons. An attempt will also be made to explain complementary information on the abacuses from that period.

II

There is clear evidence, dating back to the last third of the 10th century, that early medieval Europe began to return to the use of the abacus. The main ini-

[4] See Martianus Capella: *De nuptiis*… VI, 579, p. 204; VI, 706, p. 251; VII, 725, p. 259; resp. VII, 729, p. 261.

[5] Although the commentaries on the Martian work mention the abacus — see Remigius Autissiodorensis: *Commentum in Martianum Capellam, libri III—IX*. Ed. C.E. Lutz. Leiden 1965, p. 21.

[6] See for example W. von Speyer: *Libellus de studio*. Ed. K. Strecker. *Monumenta Germaniae Historica*. *Poetae*. T. 5/1. Leipzig 1937, pp. 169—173; or Notker der Deutsche: *Boethius, De consolatione Philosophiae*. Ed. P.W. Tax. In: *Die Werke Notkers des Deutschen*. Bd. 1. Tübingen 1986, pp. 19—20.

tiator of this revival is traditionally believed to be the Reims scholar Gerbert of Aurillac (or Reims, Bobbio, or even Ravenna), also known, from 999 until his death in 1003, as Pope Silvester II. Around the year 980, Gerbert wrote a brief treatise *Regulae de numerorum abaci rationibus*[7], which he sent to his friend Constantine at the monastery of Fleury. Moreover, not long after that (according to some interpretations, a couple of years earlier[8]) Constantine's monastic brother in Fleury, and its later abbot Abbo, referred to calculation operations on the abacus (in *In calculum Victorii commentario*)[9]. Most certainly, there were other texts dedicated to abacus calculation operations written in the first millennium, though these appear to be mostly commentaries or additions to Gerbert's *Regulae*[10]. Thus, around the year 1000, Gerbert's (probably indirect) pupils, Heriger of Lobbes (*Regulae numerorum super abacum* and *Ratio numerorum abaci*)[11] and Bernelius the younger from Paris (*Liber abaci*)[12] completed longer texts on the abacus and its use.

Apart from several anonymous treatises[13], the abacus literature during the first half of the 11th century became enriched by, for example, a treatise incor-

---

[7] Gerbertus Auriliacensis: *Regulae de numerorum abaci rationibus*. Ed. N. Bubnov. In: *Gerberti postea Silvestri II papae Opera Mathematica (972—1003)*. Berlin 1899 (repr. Hildesheim 1963), pp. 1—22. Regarding the dating see for example H.P. Lattin: *The Letters of Gerbert with his papal privileges as Sylvester II*. New York 1961, p. 46.

[8] Comp. W. Bergmann: *Innovationen im Quadrivium des 10. und 11. Jahrhunderts. Studien zur Einführung von Astrolab und Abakus im Lateinischen Mittelalter*. Stuttgart 1985, p. 180.

[9] Abbo of Fleury and Ramsey: *Commentary on the Calculus of Victorius of Aquitaine* III, 64—67. Ed. A.M. Peden. Oxford 2003, pp. 113—115; see also Abbo Floriacensis: *Excerpta ex Abbonis scolastici Floriacensis in calculum Victorii commentario*. Ed. N. Bubnov. In: *Gerberti postea Silvestri II papae Opera Mathematica*…, pp. 197—203, or: *Abbonis Abacus*. Ed. N. Bubnov. In: *Gerberti postea Silvestri II papae Opera Mathematica*…, pp. 203—204.

[10] See for example *Incertus abacista s. X: De minutiis*. Ed. N. Bubnov. In: *Gerberti postea Silvestri II papae Opera Mathematica*…, pp. 225—244.

[11] Herigerus Lobiensis: *Ratio numerorum abaci*. Ed. N. Bubnov. In: *Gerberti postea Silvestri II papae Opera Mathematica*…, pp. 221—225; comp. Herigerus Lobiensis: *Regulae de numerorum abaci rationibus*. Ed. N. Bubnov. In: *Gerberti postea Silvestri II papae Opera Mathematica*…, pp. 208—221; resp. Herigerus Lobiensis: *Regula de abaco computi*. Ed. A. Olleris. In: *Œuvres de Gerbert, pape sous le nom de Sylvestre II*. Clermont-Ferrand—Paris 1867, pp. 311—348.

[12] Bernelin, élève de Gerbert d'Aurillac: *Libre d'Abaque*. Ed. B. Bakhouche. Pau 1999.

[13] See for example *Commentarii in Gerberti regulas de numerorum abaci rationibus*. Ed. N. Bubnov. In: *Gerberti postea Silvestri II papae Opera Mathematica*…, pp. 245—284; or text wrongly atributed to Odo of Cluny — see *Regulae Domni Odonis super abacum (Opuscula de musica)*. Ed. J.-P. Migne. In: *Patrologia Latina* 133, pp. 807—814; see also *Scriti inediti relativa al calcolo dell' abaco*. Ed. B. Boncompagni. „Bulletino di bibliografia e di storia delle scienze matematiche e fisiche" 1877, no 10, pp. 595—656.

porating the first book of the so-called pseudo-Boethius's *Geometry II*[14] and by the short texts penned by Herman of Reichenau (*Qualiter multiplicationes fiant in abaco*)[15] and Laurent of Amalfi (*De divisione*)[16]. During the second half of the 11th century and in the beginning of the 12th century, the number of texts on the use of the abacus as well as the number of abacists increased. Among the most notable were Garlandus Compotista (*De abaco*)[17], Radulf of Laon (*Liber de abaco*)[18], Adelardus of Bath (*Regule abaci*)[19], and Turchillus Compotista (*Reguncule super abacum*)[20].

Earlier attempts at a chronological ordering of the oldest abacus texts[21] have been significantly revised in the recent past, thanks to the more detailed studies of the surviving depictions of the abacus from the end of the 10th as well as the 11th century[22]. Today, the prevailing opinion is that it was already at the end of the 10th century when the use of Arabic numerals (so called *ghubar*) began in Europe; however, they were only used in abacus calculations. The hypothesis concerning Gerbert's initiating role has once again increased in popularity, since Gerbert could have come into contact with *ghubar* numerals during his stay in the shadow of the Pyrenees[23]. This, in turn, constitutes the main reason as to why this article focuses primarily on the two oldest textual descriptions of the abacus written before the year 1000, or at least around this year at the latest, as well as on a couple of the oldest surviving drawings of an abacus.

---

[14] *"Boethius" Geometrie II. Ein matematisches Lehrbuch des Mittelalters* I, 19. Ed. M. Folkerts. Wiesbaden 1970, pp. 137—144.

[15] Hermannus Contractus: *Qualiter multiplicationes fiant in abaco*. Ed. P. Treutlein. "Bulletino di bibliografia e di storia delle scienze matematiche e fisiche" 1877, no 10, pp. 643—647.

[16] Laurentius Amalfitanus: *De divisione*. Ed. F. Newton. *Monumenta Germaniae Historica. Quellen zur Geistesgeschichte des Mittelalters*. T. 7. Weimar 1973, pp. 76—80.

[17] Garlandus Compotista: *De abaco*. Ed. P. Treutlein. "Bulletino di bibliografia e di storia delle scienze matematiche e fisiche" 1877, no 10, pp. 595—607.

[18] Radulphus Laudunensis: *Liber de abaco*. Ed. A. Nagl. "Abhandlungen zur Geschichte der Mathematik" 1890, Nr. 5, pp. 85—133.

[19] Adelardus Bathoniensis: *Regule abaci*. Ed. B. Boncompagni. "Bulletino di bibliografia e di storia delle scienze matematiche e fisiche" 1881, no 14, pp. 1—134.

[20] Turchillus Compotista: *Reguncule super abacum*. Ed. E. Narducci. "Bulletino di bibliografia e di storia delle scienze matematiche e fisiche" 1882, no 15, pp. 111—163.

[21] W. Bergmann: *Innovationen im Quadrivium*…, pp. 205—206.

[22] See for example M. Folkerts: *Frühe Darstellungen des Gerbertschen Abakus*. In: *Itinera mathematica*…, pp. 23—43; Idem: *The names and forms of the numerals on the abacus in the Gerbert tradition*. In: *Gerberto d'Aurillac da Abate di Bobbio a Papa dell'Anno 1000. Atti del Congresso internazionale*. Ed. F.G. Nuvolone. Bobbio 2001, pp. 245—265, or Ch. Burnett: *The Abacus at Echternach in ca. 1000 A.D.* "SCIAMVS" 2002, no 3, pp. 91—108.

[23] See Richerus Remensis: *Historiarum libri IIII* III, 43. Ed. H. Hoffmann. *Monumenta Germaniae Historica. Scriptores in Folio*. T. 38. Hannover 2000, pp. 191—192, or Ademarus Cibardi: *Historiarum libri III* III, 31. Ed. D.G. Waitz. *Monumenta Germaniae Historica. Scriptores in Folio*. T. 4. Hannover 1841, p. 130.

III

The third part of the historical treatise *Historiarium libri quattuor* by Richer of Reims, Gerbert's friend and perhaps pupil, colleague, and probably later also critic, is dedicated to the future Pope Silvester II. Richer, in his work, written during the last five years of the 10th century[24], describes in relatively great detail Gerbert's pedagogical activities in Reims, and in the individual chapters, he depicts the sources and instruments as well as Gerbert's teaching process as practised in the individual liberal arts[25]. In the closing paragraphs of his text, Richer also describes the abacus used by Gerbert.

In Reims, Gerbert created (with the help of a shield manufacturer) an instrument used to teach geometry. It was a large table divided into 27 columns (*partes*) for the insertion of jetons inscribed with written characters (*notae*, *caracteres*) of nine digits. Gerbert's abacus had thousands of usable jetons, and, judging by their positions in the required columns, it was possible to express the value of any number (for values from 10^0 to 10^{26})[26].

Richer's brief description provides the general idea of what the abacus used by Gerbert looked like. More detailed information is provided in Bernelius's treatise *Liber abaci*. Furthermore, this tract is explicitly related to Gerbert, although we know very little about its author[27]. Bernelius conceived almost the entire preface of his treatise as an ode to Gerbert's abacistic art[28] (the term *pope Gerbert* is the main argument for dating the text between the years 999 and 1003[29]), and he immediately describes the design of the computing tool in detail in the introduction to the first book.

According to *Liber abaci*, the computing tool has 30 columns (*lineae*) with the first three on the right dedicated to the calculation of fractions, and the re-

---

[24] On the author and the dating of the text see mainly J. Glenn: *Politics and History in the Tenth Century. The Work and World of Richer of Reims*. Cambridge 2004.

[25] Richerus Remensis: *Historiarum*…, III, 46—54, pp. 193—198.

[26] Ibidem, III, 54, p. 198: "Cuius introductioni, abacum id est tabulam dimensionibus aptam opere scutarii effecit. Cuius longitudini, in XXVII partibus diductę, novem numero notas omnem numerum significantes disposuit. Ad quarum etiam similitudinem, mille corneos effecit caracteres, qui per XXVII abaci partes mutuati, cuiusque numeri multiplicationem sive divisionem designarent. Tanto compendio numerorum multitudinem dividentes vel multiplicantes, ut pre nimia numerositate potius intelligi quam verbis valerent ostendi".

[27] See for example B. Bakhouche: *Introduction*. In: Bernelin, élève de Gerbert d'Aurillac: *Libre d'Abaque*…, p. 9; W. Bergmann: *Innovationen im Quadrivium*…, p. 199, U. Lindgren: *Gerbert von Aurillac und das Quadrivium. Untersuchungen zur Bildung im Zeitalter der Ottonen*. Wiesbaden 1976, p. 46.

[28] Bernelin, élève de Gerbert d'Aurillac: *Libre d'Abaque*…, pp. 16—18.

[29] See for example M. Folkerts: *The names and forms*…, p. 249.

maining 27 columns used for calculations involving whole numbers. This implies that, just as in Richer's description, the abacus here allows the expression of values of numbers from 10^0 to 10^{26}. Each group of three columns is roofed by a large arch[30] containing two other arches — a bigger one with the second and third column on the right, and a smaller arch roofing the first column in each triad[31].

Each column represents one step in the decimal numeral system. This structure, with the help of arches, was designed to guarantee easier orientation on the large table. For this very reason, the columns are marked by characters — letters explaining the values of the columns within one large arch, and thus within the three columns. Above the third column on the right, there is the letter *C* (abbreviation of *centenus* — hundred), suggesting that in this particular position one could expect to be dealing with hundreds (for example, hundreds of millions, hundreds of billions, and so on). The second column on the right is labeled with the letter *D* (abbreviation for *decenus* — tens), indicating that within the triad of columns, this one represents values of tens (for example, tens of thousands, tens of billions, and so on). Above the first column on the right, there are two letters. The first is an *M* (an abbreviation for the Greek word *monas* — unit), and since it is the mark for the units' column (*unitates*), we have, for example, thousands, quadrillions, and so on. Given that the letter *M* in descriptions of the individual gradations within an abacus often means thousands (*milia*), Bernelius accompanies the *M* indicating units in the columns with an additional letter *S* (*singulares*), stressing the fact that it signifies units[32].

Then Bernelius introduces in the abacistic board all columns designed for the calculation of integers and transfers their titles to express the individual powers of the decimal system from 10^0 to 10^{26}. He goes from right to left, and the units' column is headed with the letter *I*, the tens column is marked with the letter *X,* and there is the letter *C* for the hundreds (from the Roman numerals for one, ten and hundred); for thousands, there is the letter *I* with a dash (*titulus*) above the Roman numeral, i.e. $\bar{I}$ (in the following description, the values of thousands and millions, etc. are marked by the abovementioned $\bar{M}$), for tens of thousands, there is $\bar{X}$, etc., until the twenty-seventh column, marked $C\bar{M}$ $\bar{M}\bar{M}$ $\bar{M}\bar{M}$ $\bar{M}\bar{M}$ $\bar{M}\bar{I}$ (see tab. 2)[33].

However the abacus in Bernelius's description is not only divided vertically into 30 columns, but also horizontally, with lines, into four horizontal areas (*spatia*), described, from top to bottom, as first, second, third, and fourth[34]. As with the arches or explanatory characters above the individual columns, the

[30] Later the arches are called *arcus Pythagorei.*

[31] Bernelin, élève de Gerbert d'Aurillac: *Libre d'Abaque*…, 1, pp. 21—22.

[32] Ibidem, pp. 22—23.

[33] Ibidem, pp. 23—24.

[34] Ibidem, pp. 24—25.

horizontal structuring was designed mainly to simplify work with the abacus in order to allow various operations in various parts of the table, especially the auxiliary and complementary calculations for certain mathematical operations or to enable the use of an area for keeping running totals etc.

In compliance with Richer's description of the Gerbert abacus, Bernelius introduces the symbols for numerals (*caracteres, figurae*) used in the abacus. He always names the numerals with their Latin designation, followed by Hindu-Arabic (*ghubar*), and Greek numerals (see tab. 1)[35].

Richer's short note as well as Bernelius's description clearly refer to a very similar design or to the very same device. The only difference is that it is unlikely that Gerbert used the abacus for calculations with fractions, explained by the absence of three columns on the right. Richer does not refer to the horizontal division of the abacus nor to the characters marking the columns, though this fact could be due to the brevity of his account.

Heriger of Lobbes wrote his treatise at roughly the same time as Bernelius. Heriger does not, however, describe the abacus, but in his rules of computing, he incorporates all 27 gradations of the decimal order of magnitude as shown in tab. 2.

IV

Today, there are several surviving period images of the monastic or Gerbertian abacus. The oldest sketches of the computing tables date back to the end of the 10th century, and one of the oldest of those sketches is the so-called abacus from Echternach **[E]** (see plate 1), dating back to the late 990s[36]. Images of this abacus have survived in two manuscripts, one as a single leaf inserted into an 11th century Bible (Luxemburg, Bibliothèque nationale de Luxembourg, MS 770), and another, located in the larger manuscript from Echternach, currently displayed in Trier (Trier, Stadtbibliothek, MS 1093/1694, fol. 197r). Both abacus drawings are very similar (the Trier version does not contain the horizontal division of the abacus, and there are slight differences in the depiction of numerals and decimal scales), so we can assume the coincident origin of both drawings. The abacus from the Luxembourg Echternach has 27 columns (the

[35] Ibidem, p. 25.

[36] For description, dating and photocopy see Ch. Burnett: *The Abacus at Echternach…*, pp. 91—108.

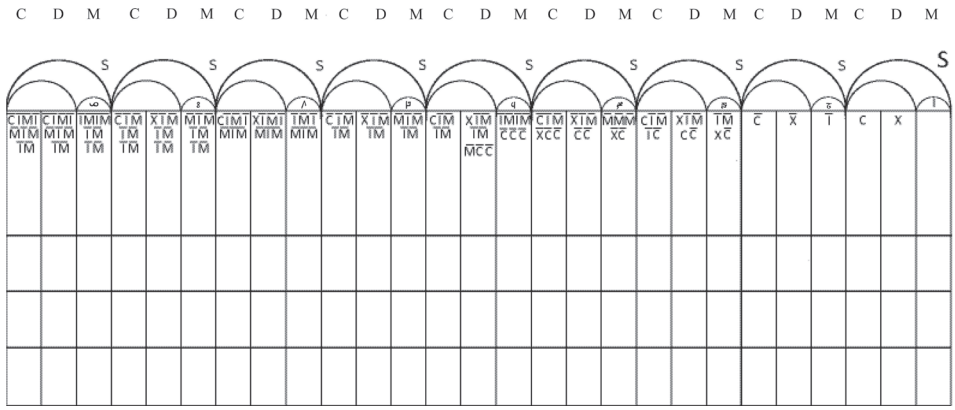


Plate 1. Abacus from Echternach **[E]** (before 1000); drawing acc. to Luxemburg, Bibliothèque nationale de Luxembourg, MS 770 and Trier, Stadtbibliothek, MS 1093/1694, fol. 197r

variant on the single leaf has been cut around the edge and the first three right columns are missing) each of them headed with the corresponding decimal scale (the entry in hundreds of thousands is written from millions, i.e. 10^6 to tens of trillions, i.e. 10^{13}). The columns end with arches, which always contain three decimal scales (from units to hundreds, thousands to hundreds of thousands, etc.). Inside of these arches, there are smaller arches helping with orientation — always a smaller one for units, thousands, millions, etc., the second, bigger one, roofing the remaining two scales within the three-column arch, thus directly corresponding to the description provided by Bernelius. All the smallest arches are marked by the letters *S* (units), and above the abacus, there are letters *C* (hundreds), *D* (tens), and *M* (units) for each triad of columns (these letters are not present in the Trier variant). Inside the smallest arches there are written the west-Arabic *ghubar* numerals, and the variant from the single leaf also divides the abacus horizontally into four parts.

During the same period, i.e. at the end of the 10th century, the abacus **[B]** from a mathematical and computation manuscript from Bern (Bern, Burgerbibliothek, MS 250, f. 1r) was drafted. This computing board (see plate 2) is perhaps the best-preserved functional abacus from the early Middle Ages. In many ways, it is similar to the **[E]** one — they share many similar characteristics, such as division into four horizontal parts, large arches roofing three columns, with one small arch for units, and a larger one for tens and hundreds as well as the *C*, *D*, and *M* characters above each triad of arches. However, there is also a number of differences[37]. The entire computing table is titled with a hexameter referring to the information that it was Gerbertus

[37] For description and image see M. Folkerts: *Frühe Darstellungen*…, p. 28, resp. p. 40 or Idem: *The names and forms*…, p. 252.

Plate 2. The Bern abacus **[B]** (end of 10th century); drawing acc. to Bern, Burgerbibliothek, MS 250, f. 1r

who introduced the abacus numerals (i.e. west-Arabic *ghubar* numerals) into the Latin world[38], though the verse does not state the shape of the numerals as in **[E]**. The most significant difference is the fact that **[B]** consists of not 27 but 30 columns, allowing calculation of the fractions using the first three columns on the right. These columns are headed with symbols for fractions (*uncia, scripulus, calcus*), and, naturally, the characters for units, tens, and hundreds (*M*, *D*, and *C*) are missing. Each of the remaining 27 columns is headed with a decimal scale (from 10^0 to 10^{26}) and similarly to **[E]**, there also appears a variant with decimal system in hundreds of thousands. However, in this particular case, the transcript is placed in the second horizontal part of the abacus and proceeds from millions to hundreds of quadrillions (10^{26}). The names, symbols and mutual ratios of the fractions (from *as* to *calcus*) are stated in the fourth bottom parallel part of the abacus. It seems that the author of the drawing had a very in-depth understanding of the skills of abacist computing.

The same cannot be said of the Paris abacus **[P]** from Fleury, preserved in the astronomy-abacist manuscript from the beginning of 11th century (Paris, Bibliothèque nationale de France, Lat. 8663, f. 49v)[39]. The author of the abacus drawing probably originally intended to depict the 27-column abacus (without the columns for fractions), as suggested by the presence of the usual triads of arches (one-, two-, and three-column ones) as well as the fact that they are marked decimally (*C*, *D*, *S* or *M*, with the character *'M'* perhaps marking thousands as the fourth scale) and titling of the individual scales. Apart from the first three columns (from units to hundreds) the author did not avoid mistakes. Instead of 24 columns for the individual decimal scales, he only drew 16, and instead of three columns, he only drew two (see plate 3). Therefore, the entire computing instrument becomes very confusing and, in fact, unusable, despite the Arabic numerals written in the smallest arches in the abacus heading.

The first surviving abacus **[G]** drawing which constitutes a part of a text treatise on abacus computing is at the same time a pictorial accompaniment to the pseudo-Boethius abacist chapter *Geometrie II*, written by an unknown author, most probably from Lorraine (see plate 4)[40]. Unlike the previous abacuses, this one has only 12 columns (from units to hundreds of billions, i.e. 10^{11}), while each scale is completed with its own small arch and in ten of these arches, there are written all west-Arabic numerals and the so-called *sipos*.

[38] See for example M. Folkerts: *Frühe Darstellungen*…, p. 28: "Gerbertus Latio numeros abacique figuras".

[39] Details and photocopy see in ibidem, pp. 28—29, resp. p. 41 or M. Folkerts: *The names and forms*…, pp. 252—253.

[40] For details, incl. handwritten photocopies see *"Boethius" Geometrie II*…, pp. 83—94, resp. Taf. 1—21.

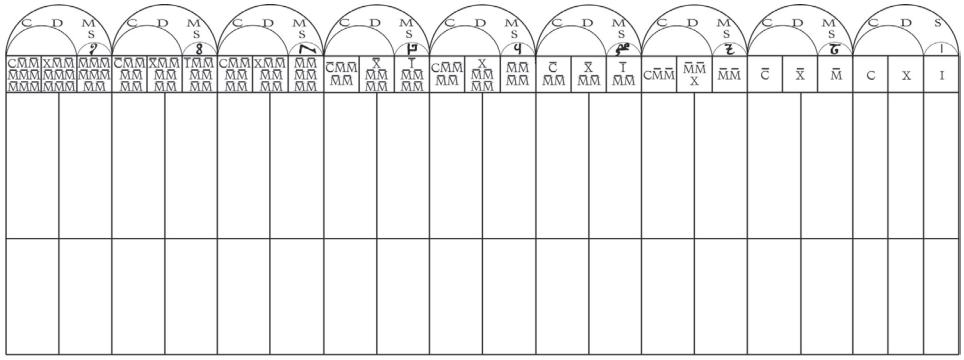


Plate 3. The Paris abacus **[P]** (beginning of 11th century); drawing acc. to Paris, Bibliothèque nationale de France, Lat. 8663, f. 49v

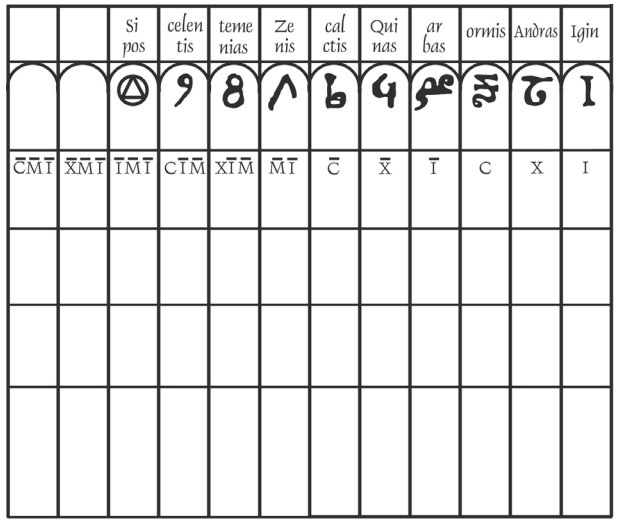


Plate 4. Abacus from so-called pseudo-Boethius *Geometry II* **[G]** (first half of 11th century); drawing acc. to Erlangen, Universitätsbibliothek 379, f. 35r

Above the arches, all ten new symbols are written in words. While **[E]** and **[P]** represent nine symbols of numerals (*de facto* directly corresponding with the first nine numerals in **[G]**), the pseudo-Boethius *Geometrie* also contains *sipos* seemingly reminiscent of our symbol for zero. However, zero was not needed in the abacus: where we would need to write zero today, for example, as in the number 507, the abacist simply left an empty column in the abacus, inserting the symbol for five (*quinas* or *quimas*) into the column for hundreds (*C*) and the symbol for the number seven (*zenis*) in the units' scale (*I*). *Sipos*, therefore, did not represent zero, but an auxiliary symbol, marking the actual place of computing in the abacus table. Furthermore, this abacus is divided

into four parts by parallel lines. The symbols of the individual decimal scales are, according to the tradition, written in the top part. In general, this drawing serves primarily as an illustration of the text, intended for the purpose of clarification for the reader of the treatise, and, in all probability, it does not represent any real computing instrument usable for mathematical operations[41].

The Vatican abacus **[V]** seems in many ways to have been inspired by the pseudo-Boethius abacus, and it constitutes part of a concluding mathematic-abacist insertion in a collection of scientific texts (Vatican, Lat. 644, f. 77v—78). Although the manuscript dates back to the 10th century, its last folios with the abacus drawing come from the 11th century[42]. Furthermore, this abacus (see plate 5), similarly to **[G]**, states names and symbols of the *ghubar* numerals (including the *sipos* assistant mark which is pushed into one column together with the number nine, probably to eliminate the possibility of confusing it with zero or ten), and it is dedicated to calculations up to hundreds of billions (i.e. 10^{11}), thus consisting of only 12 columns. It seems that the author of the drawing struggled a little (similarly to the author of **[P]**), since he subsequently added a 13th column for calculations with fractions, as evidenced in the drawing. The usual arches roofing the columns are, unusually, written into the individual columns and repeated three times. In certain ways, this abacus seems to be the most complex. Apart from columns, which include marks for the individual decimal scales (though the common division with the help of the letters *C*, *D*, and *S*/*M* is missing, similarly to **[G]**), and apart from symbols as well as names for Arabic numerals, it also includes the hexameter on Gerbert of Reims and his influence on the use of new numerals among Latin scholars. Furthermore, it contains symbols, names, and mutual ratios of fractions. The hexameter and fractions are also included in **[B]**, though in this particular case, the symbols are partially different and, more importantly, the *siliqua* is incorrectly stated (according to **[V]**, it represents 1/2 of *calcus*, while usually it would represent the value of 3/4 of *calcus*, since it is 1/6 of *scripulus*, and *calcus* is 1/8 of *scripulus*). Thus, the value of *sescuncia* is stated incorrectly as well. It is, therefore, possible to assume that the individual who drew this abacus had only partial knowledge of the mathematical issues described.

---

[41] Similar case can be found in mathematical manuscript from the 11th century (Montpellier, H 491, f. 76r, resp. f. 79r—v), including Bernelius's text *Liber abaci* with examples of abacistic calculations and Roman fractions. All this is then transferred into accompanying illustrations in form of abacistic tables.

[42] Details, incl. photocopy and references to other editions in M. Folkerts: *Frühe Darstellungen*…, pp. 29—30, resp. p. 42; or Idem: *The names and forms*…, p. 253.

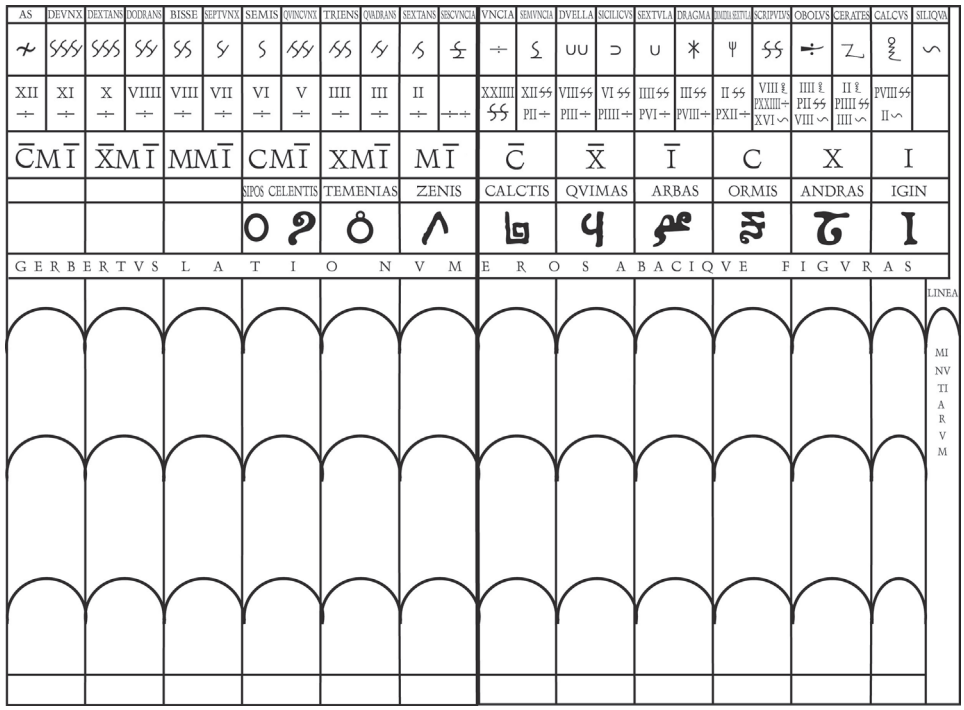


Plate 5. The Vatican abacus **[V]** (11th century); drawing acc. to Vatican, Lat. 644, f. 77v—78r

There is a number of surviving drawings of abacuses from the 11th and 12th century[43]. This article, however, will focus only on four cases. First of all, there is a manuscript from Fécamp, dating back to the 11th century and preserved in Rouen (Rouen, Bibliothèque municipale, MS 489, f. 68v—69r), which contains the drawing of an unusual, ten-column abacus **[R]** (see plate 6)[44]. Each of the columns ends with an arch with written decimal scale values, symbols, and the names of western-Arabic numerals. Apart from certain decorativeness, the abacus is particularly interesting due to the presence of Greek numerals (for values 1—9, i.e. A—Θ). As in the case of **[E]**, **[B]**, and **[G]**, this abacus is horizontally divided into four parts.

[43] For example Oxford, St. John's College, MS 17, 41v—42r. See http://digital.library.mcgill.ca/ms-17/folio.php?p=41v; resp. http://digital.library.mcgill.ca/ms-17/folio.php?p=42r.

[44] Its brief description and photocopy found in M. Folkerts: *The names and forms…*, p. 256.

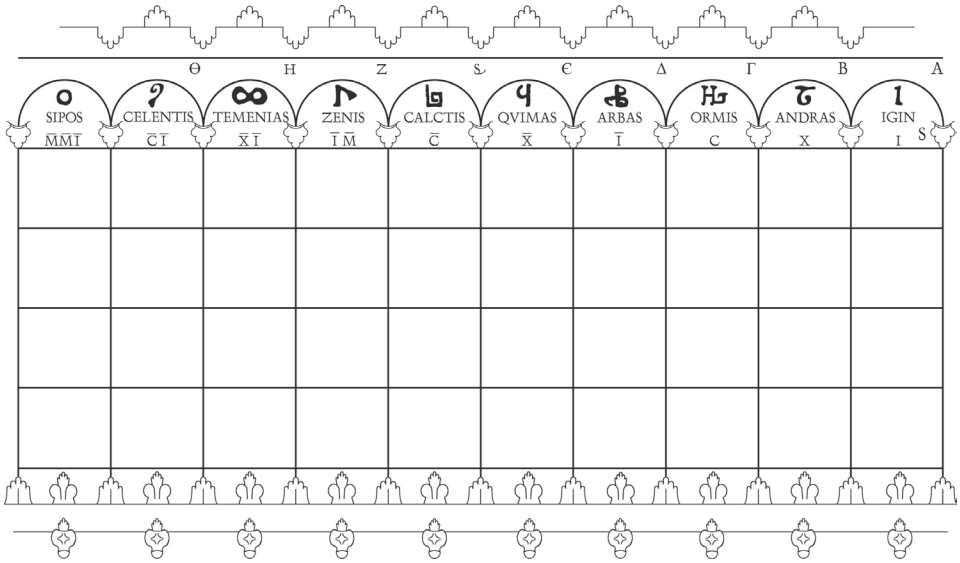


Plate 6. The Rouen abacus **[R]** (11th century); drawing acc. to Rouen, Bibliothèque municipale, MS 489, f. 68v—69r

Secondly, there is a manuscript dating back to the 12th century (Paris, Bibliothèque nationale de France, MS Lat. 7231), which summarizes texts dedicated to the liberal arts, mainly on rhetoric, and on its last folio (85v), there is a drawing of an abacus **[Pa]** (see plate 7), which dates back to the 11th century (most probably first half of the century)[45]. The reason for such an early dating is its striking similarity to the drawing of an abacus **[B]** from the late 10th century. Both drawings depict a 30-column abacus with all three sizes of arches roofing the columns with the letters *C*, *D*, and *M*/*S* (**[B]** does not include the *S siglum*), and both are horizontally divided into four sections. In the first, topmost horizontal section, in both cases, there are identically written marks for decimal scales from 10^0 to 10^{26} (abacus **[Pa]**, unlike **[B]**, does not include symbols for fractions in the first three columns on the right) and by the boundary line between the first and second horizontal sections, the scales in values of thousands are written in an identical way (in the case of **[Pa]**, they are written above the borderline, i.e. in the bottom part of the first section, while in the case of **[B]**, they are written under the dividing line, i.e. in the upper part of the second section). The fourth section in both cases represents fractions, their names (the author of the **[Pa]** abacus distorted the name of *sestertius*), symbols (very similarly, there are only tiny differences in both drawings), and mutual ratios between fractions. However, there are several significant differences between both drawings. While **[B]** states the triple numeral values of

[45] See for example Ch. Burnett: *The Abacus at Echternach*…, p. 92.

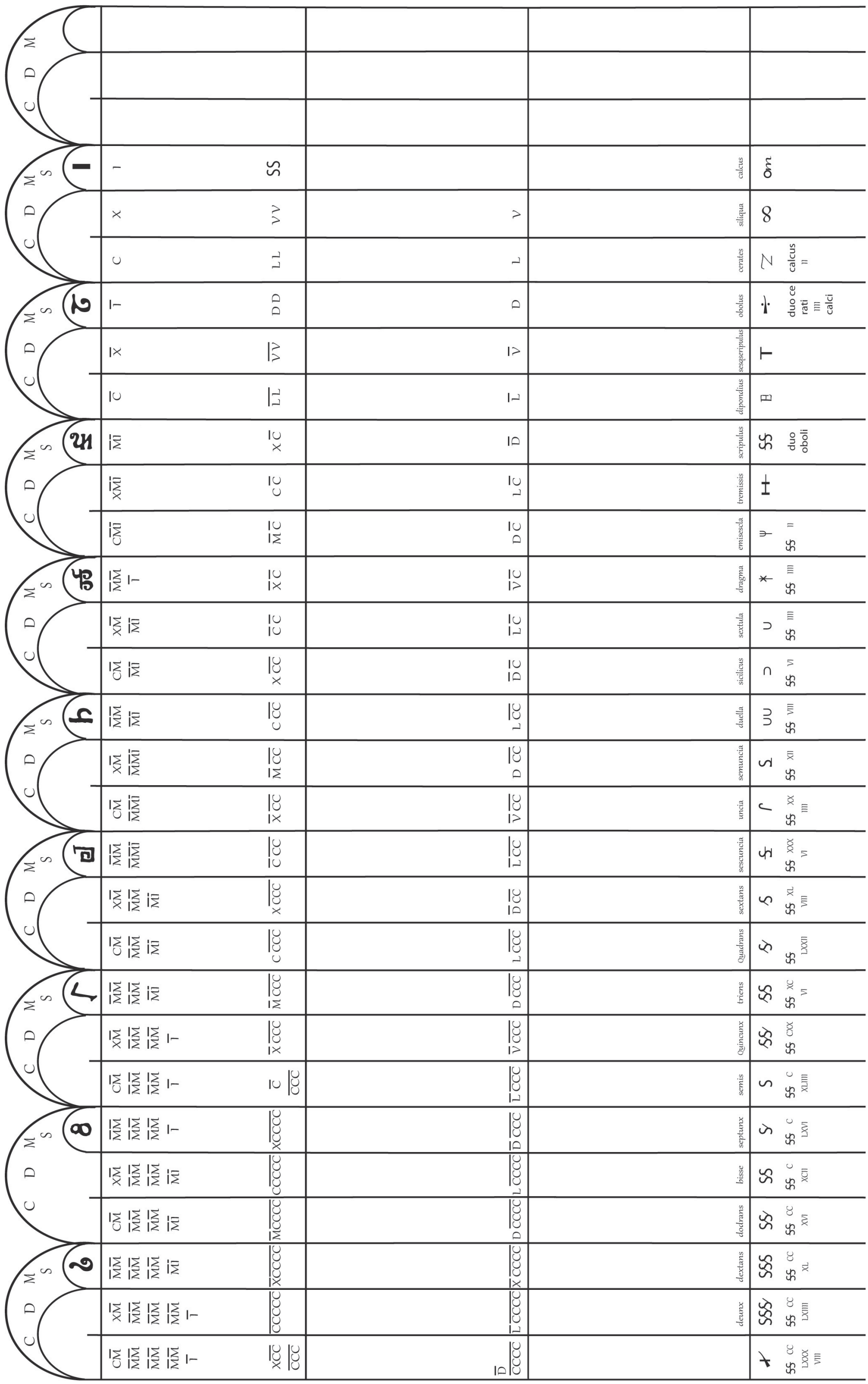


Plate 7. The Paris abacus **[Pa]** (11th century); drawing acc. to Paris, Bibliothèque nationale de France, Lat. 7231, f. 85v

as (*uncia, duella* and, on two occasions, *sicilicus*), the **[Pa]** focuses on *scripulus* and its relation to *as* or, in the case of smaller values, it adds mutual values *scripulus—obolus* and *obolus—cerates—calcus*. While abacus **[Pa]** does not include the hexameter on Gerbert's introduction of the *ghubar* numerals to the Latin West, it places the values determined by the number five inserted into the particular column, i.e. the semi-values of the decimal scales by the borderline between the second and third horizontal section. Here, the scribe made an error, since, in the third column on the left (10^{24}), he wrote the same characters as for the decimal values instead of the half-value (correctly, it should read $\bar{V}\bar{C}\bar{C}\bar{C}\bar{C}$). Furthermore, near the dividing line between the first and second horizontal sections, ahead of the markings of the columns with values of hundreds of thousands, the **[Pa]** abacus expresses the values of the columns with the help of the Roman numeral for five (scales 10^{1} to 10^{5}) and the scale of units (10^{0}) as twice the value of the *semis*. Another error in the **[Pa]** abacus is the fact that the symbols *C*, *D* and *M* are written in the first three columns on the right, dedicated to calculations with fractions and therefore devoid of the need for written decimal values. Finally, it is necessary to mention that in the smallest arches, the western-Arabic numerals are included in **[Pa]** and omitted in **[B]**. It seems, therefore, that the drawing of the **[Pa]** abacus was strongly inspired by **[B]**; however, it is not only its variation, as the author of **[Pa]** added or amended some important or interesting information, though not always in the correct way.

The last manuscript analysed in the article is a computistic manuscript from St. John's College, Oxford, dating back to the first quarter of the 12th century. It contains a particularly decorative drawing of a 27-column abacus **[J]** (Oxford, St. John's College, MS 17, 48v—49r)[46]. Of all the abacuses discussed in this article, this is the youngest, yet, at the same time, it represents a very precisely executed computing tool. Apart from the columns for calculations with fractions, it also communicates all other instructional information (see plate 8): every column is marked with the value of the decimal scale; the triad of columns (with the characters *C*, *D* and *M*/*S*) is always roofed by a large arch (as in **[E]**, **[B]**, **[P]**, and **[Pa]**) with two other smaller arches inside — one larger than the other, roofing two columns, and the second smaller arch for units (just as in Bernelius' description); in the smallest arch, the western-Arabic, Greek, and Roman numerals are inscribed with the names of Hindu-Arabic numerals above. Although the columns dedicated to calculations with fractions do not display Roman fractions, their symbols, names, and some of their ratios are written in the bottom section

[46] Abacus is available online: http://digital.library.mcgill.ca/ms-17/folio.php?p=48v; resp. http://digital.library.mcgill.ca/ms-17/folio.php?p=49r. For its analysis, interpretation incl. photocopies see G.R. Evans: *Difficillima et Ardua: theory and practice in treatises on the abacus, 950—1150*. "Journal of Medieval History" 1977, no 3, pp. 21—38; or Idem: *Schools and scholars: the study of the abacus in English Schools c. 980—c. 1150*. "The English Historical Review" 1979, no 94, pp. 71—89.

of the abacus (and in a departure from **[V]**, the *siliqua* is stated here correctly, as well as its multiples, the *bissiliqua* and the *tremissis*).

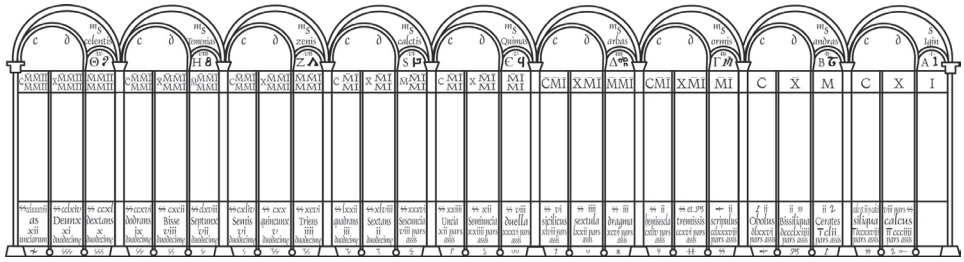


Plate 8. The Oxford abacus **[J]** (around year 1110); drawing acc. to Oxford, St. John's College, MS 17, 48v—49r

There is another schema for a computing tool in the very same manuscript (Oxford, St. John's College, MS 17, f. 35r), called *Abbonis abacus,* and, since Abbo of Fleury, known as the *abaci doctor*[47], was the contemporary of the oldest Latin pre-scholastic or early scholastic abacists, we cannot leave out this so-called Abbo abacus either (see plate 9)[48]. However, a brief glance at this drawing soon reveals that it is not a real abacus at all. Two triads of columns are roofed by arches in the "abacistic" manner, and the columns appear to represent decimal scales (from units to hundreds of thousands), a fact which is also reflected by the inscribed heading, which conveys the information that numbers up to 999,999 can be expressed[49]. However, at this point, all similarities with an abacus end. The units are placed on the left (not on the right), the individual columns contain (for the numbers written incorrectly from left to right) numeric values for each particular scale (for units 1—9, for tens 10—99 etc.), so that the entire table reminds us more of a mere list of quantity values rather than an abacus. Other typical features are also missing — units, hundreds, and thousands for triad of columns, information on the western-Arabic numerals, fractions, and so on. It is obvious that, in this case, it is not an abacistic tool.

[47] One of the preserved manuscripts of Abbo's commentaries on the Victorius's text is concluded in this manner — see Abbo of Fleury and Ramsey: *Commentary*..., p. xxxviii; see also Abbo Floriacensis: *Excerpta*…, p. 203.

[48] For its analysis, incl. photocopy see Ch. Burnett: *Abbon de Fleury abaci doctor*. In: *Abbon de Fleury. Philosophie, sciences et comput autour de l'an mil*. Ed. B. Obrist. Paris 2006, pp. 129—139 ("Oriens-Occidens. Sciences, mathématiques et philosophie de l'Antiquité à l'Age classique", no 6), resp. p. 211; for digital copy see online: http://digital.library.mcgill.ca/ms-17/folio.php?p=35r.

[49] See *Abbonis Abacus*…, p. 203: "In hac figura descriptus est numerus infinitus: incipit enim ab uno pervenitque usque ad nongentesimum millesimum". See also online: http://digital.library.mcgill.ca/ms-17/folio.php?p=35r.

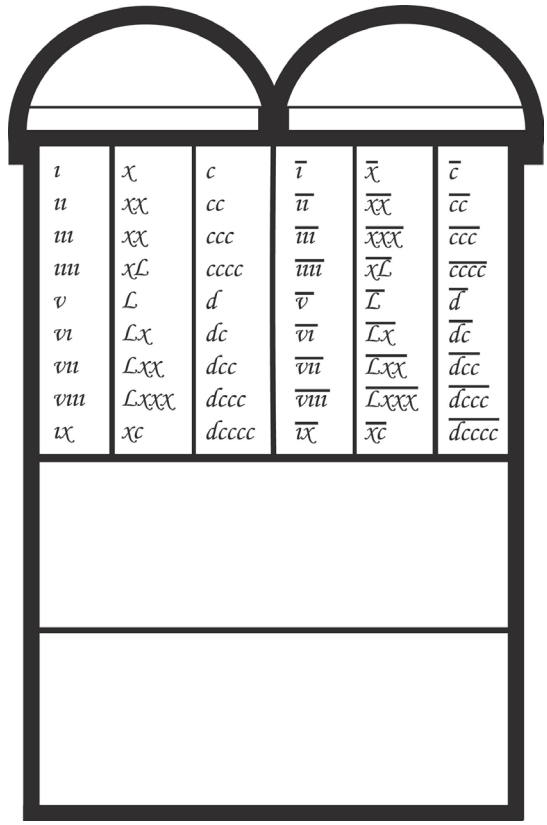


Plate 9. So-called *Abbonis abacus*; drawing acc. to Oxford, St. John's College, MS 17, 35r

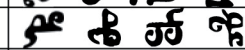
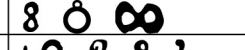
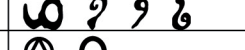
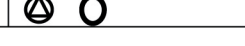
V

Despite the fact that some of the abacus drawings discussed above contain faulty information or present a computing table with a confused or fundamentally unworkable structure, it is evident that the majority of the examples correspond with Richer and Bernelius' descriptions of the tool. With the exception of the so-called Abbo abacus, it is noticeable that all diagrams of the abacus attempted to make computing practice as easily understandable as possible for the users and readers alike. However, it is clear that the authors of these dia-

grams did not always properly grasp all aspects of abacistic calculations themselves, especially the Hindu-Arabic symbols for numerals (or their Roman and Greek names and equivalents); the efforts put into the orientation markings of the individual decimal scales as well as the symbols and mutual ratios of fractions were intended to make the tool easier to use.

All in all, having examined the various depictions of the abacuses, it is possible to finally summarize the fundamental information conveyed by the oldest diagrams of abacuses, drawn in the oldest early Medieval Latin texts dedicated to the abacistic mathematics. First of all, those drawings constitute the first introduction of the western-Arabic numerals into the Western Christian world. The symbols for these numerals varied a lot, as can be observed primarily in the case of the numeral 3, as well as in the different positions of some symbols, due to the fact that the jetton with the numeral symbol could be inserted into the columns in any position, as it did not represent a serious complication for the abacist (see tab. 1).

Tab. 1. Symbols and names of numerals

Numerals	***Greek***	***Roman***	***Abacistic symbols***	***Names***
1	A	I		*igin*
2	B	II		*andras*
3	Γ	III		*ormis*
4	Δ	IIII (IV)		*arbas*
5	E	V		*quinas/quimas*
6	Σ	VI		*calctis*
7	Z	VII		*zenis*
8	H	VIII		*temenias*
9	Θ	VIIII (IX)		*celentis*
Auxiliary mark				*sipos*

Another uneasy task for the abacist was to get a better grasp of the large table and become familiar with making entries of the numeral values according to the decimal system. This task would have been eased by the headings, showing the Roman numerals of decimal scales above the individual columns. The symbols for the higher decimal scales varied a lot (see tab. 2) and could be often used incorrectly. However, the principle of marking was in fact identical. Some of the abacus drawings showed a decimal scale value in the columns not only in thousands, millions, etc., but also in hundreds of thousands.

The last frequent assistant of the abacist was an ensemble of names, symbols, and mutual ratios between fractions. Arithmetic operations with fractions

Tab. 2. Markings of the abacus columns

scale	English	Bernelius	Heriger	abacus [E]	abacus [B]	abacus [P]	abacus [J]
10^0	one	I (S, M)	I	I (S)	I (M)	I (S)	I (S)
10^1	ten	X (D)	X	X	X (D)	X (D)	X (D)
10^2	hundred	C	C	C	C	C	C
10^3	thousand	$\bar{\text{I}}$ ($\bar{\text{M}}$)	$\bar{\text{I}}$ ($\bar{\text{M}}$)	$\bar{\text{I}}$	$\bar{\text{I}}$	$\bar{\text{M}}$	M
10^4	ten thousand	$\bar{\text{X}}$	$\bar{\text{X}}$	$\bar{\text{X}}$	$\bar{\text{X}}$	$\bar{\text{X}}$	$\bar{\text{X}}$
10^5	hundred thousand	$\bar{\text{C}}$	$\bar{\text{C}}$	$\bar{\text{C}}$	$\bar{\text{C}}$	$\bar{\text{C}}$	$\bar{\text{C}}$
10^6	million	$\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{M}}\bar{\text{M}}$ ($\bar{\text{I}}$ $\bar{\text{M}}$) X $\bar{\text{C}}$	$\bar{\text{I}}$ $\bar{\text{M}}$ X $\bar{\text{C}}$	$\bar{\text{M}}\bar{\text{I}}$ X $\bar{\text{C}}$	$\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{I}}$
10^7	ten million	X$\bar{\text{M}}\bar{\text{I}}$	X $\bar{\text{M}}\bar{\text{M}}$ (X $\bar{\text{I}}$ $\bar{\text{M}}$) C $\bar{\text{C}}$	X $\bar{\text{I}}$ $\bar{\text{M}}$ C $\bar{\text{C}}$	X$\bar{\text{M}}\bar{\text{I}}$ C $\bar{\text{C}}$	X$\bar{\text{M}}\bar{\text{M}}$	X$\bar{\text{M}}\bar{\text{I}}$
10^8	hundred million	C$\bar{\text{M}}\bar{\text{I}}$	C $\bar{\text{M}}\bar{\text{M}}$ (C $\bar{\text{I}}$ $\bar{\text{M}}$) $\bar{\text{I}}$ $\bar{\text{C}}$	C $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{C}}$	C$\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{C}}$	C$\bar{\text{M}}\bar{\text{M}}$	C$\bar{\text{M}}\bar{\text{I}}$
10^9	billion	$\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}$ ($\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$) $\bar{\text{X}}$ $\bar{\text{C}}$	$\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{X}}$ $\bar{\text{C}}$	$\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{X}}$ $\bar{\text{C}}$	$\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$
10^{10}	ten billion	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{X}}$ $\bar{\text{M}}\bar{\text{M}}$ ($\bar{\text{X}}$ $\bar{\text{I}}$ $\bar{\text{M}}$) $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{X}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{C}}\bar{\text{C}}$	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{X}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{X}}\bar{\text{M}}\bar{\text{I}}$
10^{11}	hundred billion	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{C}}$ $\bar{\text{M}}\bar{\text{M}}$ ($\bar{\text{C}}$ $\bar{\text{II}}$) $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{C}}$ ($\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{C}}$)	$\bar{\text{C}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ X$\bar{\text{C}}\bar{\text{C}}$	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ X $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{C}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{C}}\bar{\text{M}}\bar{\text{I}}$
10^{12}	trillion	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ ($\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}$) $\bar{\text{X}}$ $\bar{\text{I}}$ $\bar{\text{C}}$	$\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ C$\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ C $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{I}}$
10^{13}	ten trillion	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	X $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ (X $\bar{\text{II}}$ $\bar{\text{M}}$) $\bar{\text{C}}$ $\bar{\text{I}}$ $\bar{\text{C}}$	X $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{C}}\bar{\text{C}}$	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{C}}\bar{\text{C}}$	X $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	X $\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{I}}$
10^{14}	hundred trillion	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	C $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ (C $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$), C $\bar{\text{II}}$ $\bar{\text{M}}$	C $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{X}}$ $\bar{\text{C}}\bar{\text{C}}$	C $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	C $\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{I}}$
10^{15}	quadrillion	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ ($\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$)	$\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{I}}$
10^{16}	ten quadrillion	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{X}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{X}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ X$\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{X}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{X}}$ $\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{I}}$
10^{17}	hundred quadrillion	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{C}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{C}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ C$\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{C}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{C}}$ $\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{I}}$
10^{18}	quintillion	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ ($\bar{\text{II}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$)	$\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$
10^{19}	ten quintillion	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	X $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	X $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{X}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	X $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	X $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$
10^{20}	hundred quintillion	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	C $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ (C $\bar{\text{II}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$)	C $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{C}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	C $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	C $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$
10^{21}	sextillion	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ X $\bar{\text{C}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$
10^{22}	ten sextillion	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{X}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{X}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ C $\bar{\text{C}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{X}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{X}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$
10^{23}	hundred sextillion	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{C}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{C}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{C}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{C}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{C}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$
10^{24}	septillion	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	$\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{X}}$ $\bar{\text{C}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	$\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	$\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$
10^{25}	ten septillion	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	X $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	X $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	X$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ $\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	X $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	X $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$
10^{26}	hundred septillion	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$	C $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$	C $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$ $\bar{\text{I}}$ $\bar{\text{M}}$	C$\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{M}}\bar{\text{I}}$ X$\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$ $\bar{\text{C}}\bar{\text{C}}$	C $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{M}}\bar{\text{M}}$	C $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$ $\bar{\text{M}}\bar{\text{M}}$ $\bar{\text{II}}$

Tab. 3. Fractions according to drawings of early medieval abacuses

	as	***uncia***	***scripulus***	***calcus***
as	1	12	288	2304
deunx	11/12	11	264	2112
dextans	5/6	10	240	1920
dodrans	3/4	9	216	1728
bisse	2/3	8	192	1536
septunx	7/12	7	168	1344
semis	1/2	6	144	1152
quinqunx	5/12	5	120	960
triens	1/3	4	96	768
quadrans	1/4	3	72	576
sextans	1/6	2	48	384
sexcuncia	1/8	3/2	36	288
uncia	1/12	1	24	192
semiuncia	1/24	1/2	12	96
duella	1/36	1/3	8	64
sicilicus	1/48	1/4	6	48
dragma (sextula)	1/72	1/6	4	32
(h)emisescla (dimidia sextula)	1/144	1/12	2	16
tremissis	1/216	1/18	3/2	12
scripulus	1/288	1/24	1	8
obolus	1/576	1/48	1/2	4
bissiliqua	1/864	1/72	1/3	8/3
cerates	1/1152	1/96	1/4	2
siliqua	1/1728	1/144	1/6	4/3
calcus	1/2304	1/192	1/8	1

were, along with division, regarded as a high-level skill, which could be mastered only by an experienced mathematician. The lack of this skill was to be partially rectified with the help of written overviews placed in the bottom part of the table (see tab. 3). Without any doubt, abacistic computing represented a significant change in fundamental arithmetic operations. It was not easy for

the intellectuals of that period to become familiar with the subject of abacism. Therefore, the oldest Latin texts dedicated to the abacus, along with the oldest abacus drawings, offered, in many cases, instructional information, in order to clarify how to work with an abacus as much as possible.

Marek Otisk

Opisy i przedstawienia wczesnośredniowiecznego łacińskiego abakusa

Streszczenie

W niniejszym artykule omówiono abakus z przedscholastycznego okresu średniowiecza. Na podstawie dwóch opisów pochodzących z ok. 1000 roku (jeden zawarty jest w trzecim tomie *Historii* autorstwa Richera z Reims, drugi zaś w pierwszym tomie *Liber Abaci* autorstwa Berneliusa młodszego z Paryża), głównie zaś opierając się na dziewięciu przedstawieniach tego przyrządu do liczenia, jakie zachowały się w manuskryptach z okresu od końca X do początku XII wieku (tzw. abakus z Echternach i abakus z Bern — każdy z nich z końca X wieku; abakus z Paryża z początku XI wieku; abakus z dzieła tzw. Pseudo-Boecjusza, zatytułowanego *Geometria II*, z połowy XI wieku; abakus z Watykanu, abakus z Rouen oraz abakus z Paryża — wszystkie z XI wieku; abakus z Oxfordu z początku XII wieku oraz tzw. abakus Abbona z Fleury), autor szczegółowo prezentuje wspomniany przyrząd arytmetyczny. W głównej mierze skupia się na odtworzeniu jego formy na podstawie zachowanych przedstawień oraz ich porównania. Szczegółowo opisuje i analizuje poszczególne części abakusa (kolumny, łuki itp.), a ponadto prezentuje i wyjaśnia dodatkowe informacje matematyczne pojawiające się we wspomnianych już przedstawieniach tego przyrządu w manuskryptach (np. symbole rachmistrzowskie oraz nazwy liczebników, oznaczenia kolumn abakusa oraz symbole ułamków i relacje między nimi).

Marek Otisk

Die Beschreibungen und die Darstellungen vom frühmittelalterlichen lateinischen Abakus

Zusammenfassung

Der vorliegende Artikel ist dem aus der vorscholastischen Zeit des Mittelalters stammenden Abakus gewidmet. Anhand der zwei Beschreibungen aus dem Jahr 1000 (die eine befindet sich im dritten Band der *Historiae* Richers von Reims, die andere im ersten Band des *Libre d'Abaque* des Bernelin von Paris) aber vor allem aufgrund der neun Darstellungen des Rechenhilfsmittels, die in den Manuskripten aus dem Zeitraum vom Ende des 10. bis zum Anfang des 12.Jhs erhalten geblieben sind (sog. Abakus von Echternach und Abakus von Bern — jeder vom Ende des 10.Jhs; Abakus von Paris vom Anfang des 11.Jhs; Abakus aus dem Werk *Geometrie II.* des sog. Pseudo-Boethius; Abakus von Vatikan; Abakus von Rouen und Abakus von Paris — alle vom 11.Jh.; Abakus von Oxford vom Anfang des 12.Jhs und sog. Abakus des

Abbo von Fleury) präsentiert der Verfasser detailliert das arithmetische Gerät. Er konzentriert sich vor allem darauf, die Form des Abakus anhand der erhalten gebliebenen Darstellungen zu schildern und miteinander zu vergleichen. Der Verfasser beschreibt ausführlich und analysiert die einzelnen Elemente des Abakus (Kolumnen, Bögen u.dgl.) und erklärt die in den genannten Manuskripten erscheinenden zusätzlichen mathematischen Informationen (z.B.: Rechensymbole und Bezeichnungen für Numeralien, Bezeichnungen für die Kolumnen, Bruchzahlsymbole und die Wechselbeziehungen zwischen ihnen).