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Report of Meeting

The Fifteenth Katowice–Debrecen Winter Seminar Będlewo (Poland), January 28–31, 2015

The Fifteenth Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities was held in the Mathematical Research and Conference Center Będlewo, Poland, from January 28 to 31, 2015. It was organized by Stefan Banach International Mathematical Center.

14 participants came from the University of Debrecen (Hungary), 13 from the University of Silesia in Katowice (Poland) and one from each of the following universities: University of Miskolc (Hungary), Pedagogical University of Cracow, Kraków (Poland) and Vologda State University, Vologda, (Russian Federation)

Professor Roman Ger opened the Seminar and welcomed the participants to Będlewo.

The scientific talks presented at the Seminar focused on the following topics: equations in a single variable and in several variables, iteration theory, equations on abstract algebraic structures, regularity properties of the solutions of certain functional equations, functional inequalities, Hyers–Ulam stability, functional equations and inequalities involving mean values, generalized convexity. Interesting discussions were generated by the talks.

There was also a Problem Session and a festive dinner.

The closing address was given by Professor Zsolt Páles. His invitation to hold the Sixteenth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities in February 2016 in Hungary was gratefully accepted.

Summaries of the talks in alphabetic order of the authors follow in section 1 and the list of participants in the second section.

1. Abstracts of talks

Michał Baczyński: *On the exchange principle for R-implications*

The exchange principle (EP), i.e., the equation of the form

$$I(x, I(y, z)) = I(y, I(x, z))$$

where $I\colon [0,1]^2 \to [0,1]$, generalizes the classical tautology

$$p \to (q \to r) \equiv q \to (p \to r)$$

and is the one of most important properties of a fuzzy implication both from theoretical and applicational point of view (see [1]). Furthermore, the exchange principle is closely related to the permutability equation

$$F(F(a,b),c) = F(F(a,c),b),$$

for which new results have been recently obtained in [2].

In our talk we discuss this equation (EP) in the family of R-implications, which is one of the most established classes of multivalued implications.

Definition 1 (see [4]). An associative, commutative and non-decreasing operation $T\colon [0,1]^2 \to [0,1]$ is called a t-norm if it has the neutral element 1.

Definition 2 (see [1]). A function $I\colon [0,1]^2 \to [0,1]$ is called an R-implication, if there exists a t-norm T such that

$$I(x,y) = \sup\{t \in [0,1] \mid T(x,t) \le y\},$$

for all $x, y \in [0,1]$. If an R-implication is generated from a t-norm T, then we will often denote it by I_T.

It is well-known that the residual I_T of a left-continuous t-norm T satisfies the exchange principle (EP) for all $x, y, z \in [0,1]$ (see [1]). However, the left-continuity of T is only sufficient and not necessary, as many examples in the literature illustrate. We present some necessary and sufficient conditions on a t-norm for its residual to satisfy (EP) (cf. [3]).

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Karol Baron: *On the continuous dependence in the problem of a convergence of iterates of random-valued functions*

Let $(\Omega, \mathcal{A}, P)$ be a probability space and (X, ϱ) be a complete and separable metric space.

Denote by $\mathcal{B}$ the σ–algebra of all Borel subsets of X. We say that $f\colon X \times \Omega \to X$ is a *random–valued* function (an *rv–function* for short) if it is measurable with respect to the product σ–algebra $\mathcal{B} \otimes \mathcal{A}$. The iterates of such an rv–function are given by (cf. [2, Section 1.4])

$$f^1(x, \omega_1, \omega_2, \ldots) = f(x, \omega_1), \quad f^{n+1}(x, \omega_1, \omega_2, \ldots) = f(f^n(x, \omega_1, \omega_2, \ldots), \omega_{n+1})$$

for x from X and $(\omega_1, \omega_2, \ldots)$ from Ω^∞ defined as $\Omega^{\mathbb{N}}$. Note that $f^n\colon X \times \Omega^\infty \to X$ is an rv–function on the product probability space $(\Omega^\infty, \mathcal{A}^\infty, P^\infty)$.

Consider the set $\mathcal{R}_c$ of all rv-functions $f : X \times \Omega \to X$ such that

$$\int_\Omega \varrho(f(x,\omega), f(z,\omega)) P(d\omega) \le \lambda_f \varrho(x,z) \quad \text{for } x, z \in X$$

with a $\lambda_f \in [0,1)$, and

$$\int_\Omega \varrho(f(x,\omega), x) P(d\omega) < \infty \quad \text{for } x \in X.$$

If $f \in \mathcal{R}_c$ and $\pi_n^f(x,\cdot)$ denote the distribution of $f^n(x,\cdot)$, then according to [1, Theorem 3.1] there exists a distribution π^f on X (i.e., a probability measure defined on $\mathcal{B}$) such that for every $x \in X$ the sequence $(\pi_n^f(x,\cdot))_{n\in\mathbb{N}}$ converges weakly to π^f. Consequently, we have an operator

$$f \mapsto \pi^f, \quad f \in \mathcal{R}_c,$$

and we are interested in a kind of its continuity.

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Mihály Bessenyei: *The contraction principle in extended form*

There are several extensions of the classical Banach Fixed Point Theorem in technical literature. A branch of generalizations replaces usual contractivity by weaker but still effective assumptions. The talk is devoted to (nonlinear) quasicontractions, presenting an elementary proof for a known fixed point theorem. Some partial results in semimetric spaces are also investigated.

Zoltán Boros: *Continuity of convex functions and linear functionals* (Joint work with Zsolt Páles)

Ger and Kominek proved [2] that, for any subset T of a real linear topological Baire space X, the following implications are equivalent:

(i) If $\phi\colon X \to \mathbb{R}$ is additive and bounded from above on T, then ϕ is continuous.
(ii) Suppose that $T \subset D \subset X$, D is open and convex. If $f\colon D \to \mathbb{R}$ is Jensen-convex and bounded from above on T, then f is continuous.

Applying a former result [1] of the present authors, we can separately investigate analogous questions for the linearity of additive functionals and the convexity of Jensen-convex functions, which can be established for an arbitrary real linear space X (with a reasonable assumption on D). We also investigate the role of the Baire property of the corresponding topology when we consider the continuity of these functionals.

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Pál Burai: *Functional equations involving Makó–Páles means* (Joint work with Justyna Jarczyk)

In this talk we investigate some functional equations involving Makó–Páles means.

Szymon Draga: *On a problem of Zoltán Boros, II* (Joint work with Janusz Morawiec)

We present a few results on reducing the order of the polynomial-like iterative equations $\sum_{n=0}^{N} a_n g^n(x) = 0$, where $g\colon \mathbb{R} \to \mathbb{R}$ is the unknown continuous function. As an application, we obtain an answer to a problem posed by Zoltán Boros (during the 50th International Symposium on Functional Equations, 2012) of determining all continuous functions $f\colon (0,+\infty) \to (0,+\infty)$ satisfying $f^n(x) = \frac{[f(x)]^n}{x^{n-1}}$, where $n \geq 3$ is an arbitrary integer number.

Włodzimierz Fechner: *Some remarks on the Maligranda–Orlicz inequality*

A φ-function is a nondecreasing continuous map $f\colon [0,+\infty) \to [0,+\infty)$ such that $f(u) = 0$ if and only if $u = 0$ and $\lim\limits_{t\to+\infty} f(t) = +\infty$.

In 1987 Lech Maligranda and Władysław Orlicz proved that if f is a convex φ-function, n is a positive integer, a_k are arbitrary non-negative numbers for $k = 1,\dots,n$ and $a_0 = 0$, then

$$\sum_{k=1}^{n} |f(a_k) - f(a_{k-1})| \leq f\left(\sum_{k=1}^{n} |a_k - a_{k-1}|\right). \tag{1}$$

Inequality (1) is called Maligranda–Orlicz inequality.

In 1996 Josip Pečarić and Ivica Gusić proved (1) for functions defined on the set $[0,\beta_1) \times \cdots \times [0,\beta_r)$ for some $\beta_1,\dots,\beta_r \in (0,+\infty]$. Two years later Gusić proved this inequality for mappings acting between positive cones of lattice-ordered groups.

Our goal is to complement some of the results of Gusić. We will solve two functional inequalities introduced by Gusić, which are related to (1).

Roman Ger: *Mild regularity of convex differences*

Given a real function f defined on an open interval $(a,b) \subset \mathbb{R}$ we deal with a family $\{g_{\lambda,y} : \lambda \in (0,1),\, y \in (a,b)\}$ of convex differences $g_{\lambda,y} : (a,b) \longrightarrow \mathbb{R}$ given by the formula

$$g_{\lambda,y}(x) := \lambda f(x) + (1-\lambda) f(y) - f\left(\lambda x + (1-\lambda) y\right), \quad x \in (a,b).$$

We look for regularity conditions upon $g_{\lambda,y}$ forcing f to be convex. Getting them we carry over these results to mappings f with values in a suitable Banach lattice.

Attila Gilányi: *On subquadratic functions*

In this talk, we present some results on (weakly) subquadratic functions, i.e., on the solutions of the functional inequality

$$f(x+y)+f(x-y)\leq 2f(x)+2f(y), \quad x,y\in G,$$

in the class of real valued functions defined on a group $G=(G,+)$ with the aim of pointing out some possible generalizations of those theorems for submonomial functions.

Dávid Csaba Kertész: *The 'Isabelle' proof assistant software*

Proof assistants are computer programs that help developing mathematical theories by human-machine interaction. They check the correctness of proofs and some can even prove theorems automatically. A theory written in a proof assistant software is guaranteed to be correct. Among the available proof assistants, 'Isabelle' has some outstanding features. The 'Isabelle' code is designed to be readable both by humans and the machine. It can make usual LaTeX source code from the theories, and it is highly capable of proving theorems automatically. In this talk we present the capabilities and possible applications of 'Isabelle' in more detail.

Tibor Kiss: *Asymmetrical M-convexity of extended real valued functions* (Joint work with Zsolt Páles)

If $I\subseteq\mathbb{R}$ is an interval and $f\colon I\to\mathbb{R}$ is a given function, then let $\mathcal{C}_f$ be the set of all parameters t from $[0,1]$, for which the function f is t-convex in the standard sense. It is easy to see, that this set can not be empty. Kuhn's theorem (cf. [3, Theorem 1K]) states, that we have two cases, namely $\mathcal{C}_f=\{0,1\}$ or there exists a subfield F of $\mathbb{R}$, such that $\mathcal{C}_f=[0,1]\cap F$.

The aim of my talk is to mention some properties of this set concerning asymmetrical M-convexity of extended real valued or real valued functions. As special case we obtain algebraic properties of $\mathcal{AC}_f$, i.e. some properties of the family of parameters concerning asymmetrical convexity.

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Katarzyna Kuhlmann: *Ball Spaces and Fixed Point Theorems* (Joint work with Franz-Viktor Kuhlmann)

Ball spaces provide a general framework for the proof of fixed point theorems in several different settings (metric, ultrametric, ordered abelian groups and fields, topological spaces, posets, lattices, etc.). We present general FPTs which then can be specialized to FPTs in several different situations by choosing the "balls" (the distinguished sets) according to the application we have in mind. Having a common denominator for the various applications also allows us to transfer known results from one application to another where they had not been previously observed.

Rezső L. Lovas: *On an exotic topology of the integers* (Joint work with István Mező)

I present some basic properties of the Fürstenberg topological space and three different metrics on the set of integers inducing the Fürstenberg topology. Then I construct its metrical completion by using the factorial number system. I also show that this completion is a natural construction in the sense that the completions of any two translation invariant metrizations of this topology are uniformly equivalent.

Radosław Łukasik: *Stability of inner product preserving mappings*

For Hilbert spaces X and Y over the same field $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, we study the class of mappings $f, g\colon X \to Y$ preserving approximately inner product in the following sense:

$$|\langle f(x)|g(y)\rangle - \langle x|y\rangle| \le \varphi(x,y), \quad x, y \in X \setminus \{0\}, \tag{1}$$

where $\varphi\colon X\times X \to [0,\infty)$ is some function. We show that there exist $F, G\colon X \to Y$ near resp. f and g such that

$$\langle F(x)|G(y)\rangle = \langle x|y\rangle, \quad x, y \in X. \tag{2}$$

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Judit Makó: *On Wright convex type inequalities* (Joint work with Pál Burai and Attila Házy)

Let $F\colon \mathbb{R}^2 \to \mathbb{R}$ be a given function and let D be a nonempty open convex subset of a normed space X. We say that $f\colon D \to \mathbb{R}$ is F-Wright convex, if

$$F(f(tx+(1-t)y), f((1-t)x+ty)) \leq F(f(x), f(y)), \quad x, y \in D,\ t \in [0,1].$$

If the above inequality stands only one $t \in]0,1[$, we say that f is (t,F)-Wright convex. In this talk, we give some basic properties of F-Wright convex functions, we prove a Bernstein–Doetsch type theorem, moreover we will look for connections between F-Wright convex functions and Hermite–Hadamard type inequalities.

Gyula Maksa: *On the alienation of the multiplicative Cauchy equation and the Hosszú equation*

This is a supplement to the work joint with Roman Ger and Maciej Sablik I presented in the previous DKWS in Hajdúszoboszló.

Let $g, h\colon [0,1] \to \mathbb{R}$ and $\Gamma(x,y) = g(x)g(y) - g(xy)$, $x, y \in [0,1]$. Supposing that h is differentiable and Γ is not identically zero, we solve the functional equation

$$\text{(1)} \quad g(x)g(y) - g(xy) = h(x+y-xy) - h(x) - h(y) + h(xy), \quad x, y \in [0,1].$$

It follows from the result that (1) is not equivalent with the system

$$g(x)g(y) = g(xy), \quad x, y \in [0,1],$$
$$h(x+y-xy) + h(xy) = h(x) + h(y), \quad x, y \in [0,1],$$

that is, the 'alienation' does not hold even in this case.

A conjecture and open problems will also be presented.

Janusz Morawiec: *On a problem of Zoltán Boros, I* (Joint work with Szymon Draga)

We determine all continuous solutions $g\colon I \to I$ of the polynomial-like iterative equation $g^3(x) = 3g(x) - 2x$, where $I \subset \mathbb{R}$ is an interval. In particular, we obtain an answer to a problem posed by Zoltán Boros (during the 50th International Symposium on Functional Equations, 2012) of determining all continuous functions $f\colon J \to J$, where $J \subset (0, +\infty)$ is an interval, satisfying $f^3(x) = \frac{[f(x)]^3}{x^2}$.

Gergő Nagy: *Resolving sets in metric spaces*

Concerning certain investigations related to metric spaces it may be useful to find such sets which generate these spaces in a certain metric sense. The resolving sets are particular examples of sets with this property, they are those subsets of a metric space X for which any element of X can be uniquely identified by its distances to the points of such a subset. The metric dimension of X is the minimum κ of the cardinality of all resolving sets in X. In this talk after presenting some general results concerning the metric dimension, we are going to show several statements related to resolving sets in certain subspaces of $\mathbb{R}^n$ and to the metric dimensions of the subspaces under consideration. The last part of the talk is devoted to theorems on resolving sets in particular metric spaces of self-adjoint operators.

Andrzej Olbryś: *On separation theorems for delta-subadditive and delta-superadditive mappings*

Let $(X, +)$ be an abelian group, and let $(Y, \|\cdot\|)$ be a real Banach space. Motivated by the dissertation of L. Veselý and L. Zajiček [3] R.Ger in [2] considered the following functional inequality

$$\|F(x) + F(y) - F(x+y)\| \leq f(x) + f(y) - f(x+y), \quad x, y \in X.$$

If a pair (F, f) satisfies the above inequality then we say that a map $F\colon X \to Y$ is delta-subadditive with a control function $f : X \to \mathbb{R}$. If a pair $(-G, -g)$ satisfies the above inequality, then we say that G is delta-superadditive with a control function g. Inspired by methods contained in [1] we generalize the well known separation theorems for subadditive and superadditive functionals to the case of delta-subadditive and delta-superadditive mappings. We also consider the problem of supporting delta-subadditive maps by additive ones. As a consequence of these theorems we obtain some stability results for Cauchy's equation.

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Zsolt Páles: *On the characterization of differences of ω-convex functions*

In the talk we will introduce the notion of a second-order variation which enables us to characterize functions that are differences of ω-convex functions, where $\omega = (\omega_1, \omega_2)$ is a two-dimensional Chebyshev system.

Bella Popovics: *Convexity without convex combination* (Joint work with Mihály Bessenyei)

To define a classical convex set or function we should suppose that the basic space has a sort of linear structure. There exist many extensions of convexity which do not use linearity, for example applying the continuity and unique interpolation property of Beckenbach families we can define generalized convex sets and functions. In this extension some basic separation problems can be proved. The question is what happens if the continuity is also left. The aim of the talk is to present such a minimal system of axioms which implies convexity, and to show those theorems that hold in this general case.

Ekaterina Shulman: *On stability of the Levi–Civita equation on amenable groups*

We consider the functions on an amenable group G which are "approximate solution" of the Levi–Civita functional equation

$$f(gh) = \sum_{j=1}^{N} u_j(g)v_j(h), \quad g, h \in G,$$

and try to prove that they are "close" to proper solutions. For bounded functions the problem was solved earlier [1] by means of the techniques of covariant widths of convex compacts in Banach G-spaces. Here we show how the general case can be studied by estimation of norms of operators that realize 1-cocycles of representations of G on Banach spaces.

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Justyna Sikorska: *Approximately orthogonally additive set-valued mappings*

It is known that a set-valued orthogonally additive function from an orthogonality space into a family of all nonempty compact and convex subsets of a Fréchet space is of the form $a + Q$, where a is single-valued additive and Q is set-valued quadratic (J. Sikorska [3]).

We will study approximately orthogonally additive set-valued mappings. We recall here that the single-valued orthogonal Cauchy equation is stable (R. Ger, J. Sikorska [2], W. Fechner, J. Sikorska [1]).

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Éva Székelyné Radácsi: *Chebyshev system and convexity over finite sets* (Joint work with Zsolt Páles)

Let I be a nonempty real interval and let $(\omega_1, \dots, \omega_n)$ be a positive n-dimensional Chebyshev system. A function $f\colon I \to \mathbb{R}$ is called $(\omega_1, \dots, \omega_n)$-convex over the elements $x_0 < x_1 < \dots < x_n$ in I if

$$\Omega(x_0, x_1, \dots, x_n; f) := \begin{vmatrix} \omega_1(x_0) & \omega_1(x_1) & \dots & \omega_1(x_n) \\ \dots & \dots & \dots & \dots \\ \omega_n(x_0) & \omega_n(x_1) & \dots & \omega_n(x_n) \\ f(x_0) & f(x_1) & \dots & f(x_n) \end{vmatrix} \geq 0.$$

Let $x_0 < x_1 < \dots < x_m$ be some points in I, $(m > 2)$ and let (ω_1, ω_2) be a positive 2-dimensional Chebyshev system on I. Using the connection of (ω_1, ω_2)-convexity to ordinary convexity, one can see that if a function $f\colon I \to \mathbb{R}$ is (ω_1, ω_2)-convex for all $x_{i-1} < x_i < x_{i+1}$, $i \in \{1, \dots, m-1\}$, then f is (ω_1, ω_2)-convex for all $x_j < x_k < x_l$, where $j < k < l$ and $j, k, l \in \{0, \dots, m\}$ hold, i.e.

$$\Omega(x_j, x_k, x_l; f) \geq 0.$$

The aim is to generalize the above result by showing that, if $(\omega_1, \dots, \omega_n)$ is a positive n-dimensional Chebyshev system and the function $f\colon I \to \mathbb{R}$ is $(\omega_1, \dots, \omega_n)$-convex for all consecutive $n+1$ points of a finite subset S of I, then f is $(\omega_1, \dots, \omega_n)$-convex over all $n+1$ points of S. Consequences of this result will also be discussed.

Patrícia Szokol: *Transformations preserving norms of means of positive operators and nonnegative functions* (Joint work with Lajos Molnár)

Motivated by recent investigations on norm-additive and spectrally multiplicative maps on various spaces of functions, in this presentation we determine all bijective transformations between the positive cones of standard

operator algebras over a Hilbert space which preserve a given symmetric norm of a given mean of elements. (We note that by a standard operator algebra we mean a subalgebra of $B(H)$ the algebra of all bounded linear operators on H which contains all finite rank operators in $B(H)$. Furthermore, we say that the norm N on $B(H)$ is a symmetric norm, if $N(AXB) \leq \|A\|N(X)\|B\|$ holds for all $A, B, X \in B(H)$.)
A result of similar spirit is also presented concerning transformations between cones of nonnegative elements of certain algebras of continuous functions.

TOMASZ SZOSTOK: *Levin-Stechkin theorem and inequalities of the Hermite–Hadamard type*

In [2] Ohlin lemma on convex stochastic ordering was used to obtain inequalities of the Hermite–Hadamard type. In this talk we use a result of Levin and Stechkin from [1] which, in fact, generalizes the result of Ohlin. Using this theorem we obtain inequalities of the forms:

$$\sum_{i=1}^{3} a_i f(\alpha_i x + (1-\alpha_i)y) \leq \frac{1}{y-x}\int_x^y f(t)dt,$$
$$a_1 f(x) + \sum_{i=2}^{3} a_i f(\alpha_i x + (1-\alpha_i)y) + a_4 f(y) \geq \frac{1}{y-x}\int_x^y f(t)dt,$$

and

$$af(\alpha_1 x + (1-\alpha_1)y) + (1-a)f(\alpha_2 x + (1-\alpha_2)y) \leq b_1 f(x) + b_2 f(\beta x + (1-\beta)y) + b_3 f(y),$$

which are satisfied by all convex functions $f\colon [x,y] \to \mathbb{R}$. As it is easy to see, the same methods may be applied to deal with longer expressions of the forms considered. As particular cases of our results we obtain some known inequalities.

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ADRIENN VARGA: *On the characteristic polynomials of linear functional equations* (Joint work with Csaba Vincze)

The solutions of a linear functional equation are typically generalized polynomials. The existence of the non-trivial monomial terms strongly depends on the algebraic properties of some related families of parameters. In extremal cases (the parameters are algebraic numbers or the parameters form an algebraically independent system) we have elegant methods to decide the existence of non-trivial solutions. We are going to extend and unify the treatment of the existence problem by introducing the characteristic polynomials of a linear functional equation. The algebraic properties of the roots allows us to conclude the existence of non-trivial solutions.

Paweł Wójcik: *Characterizations of inner product spaces by some inequalities*

Using the notion of semi-inner product in normed spaces, in this report we show some new characterizations of inner product spaces. We answer a question posed by Dragomir, whether the property (N) is characteristic for inner product spaces. We show that the space X is of (N)-type if and only if the norm in X comes from an inner product.

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