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Report of Meeting

The Eleventh Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities, Wisła-Malinka (Poland), February 2–5, 2011

The Eleventh Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities was held in the Wisła-Malinka, Poland, from February 2 to 5, 2011. It was organized by the Institute of Mathematics of the Silesian University from Katowice.

30 participants came from the University of Debrecen (Hungary) and the Silesian University of Katowice (Poland) at 15 from each of both cities. The 31^{st} participant of the Seminar was Professor Peter Volkmann from the Karlsruhe Institute of Technology who is at present a visiting professor at the Silesian University of Katowice.

Professor Roman Ger opened the Seminar and welcomed the participants to Wisła-Malinka.

The scientific talks presented at the Seminar focused on the following topics: equations in a single variable and in several variables, iteration theory, equations on algebraic structures, regularity properties of the solutions of certain functional equations, functional inequalities, Hyers–Ulam stability, functional equations and inequalities involving mean values, generalized convexity. Interesting discussions were generated by the talks.

There was also a Problem Session.

The social program included a festive dinner and an excursion which consisted of a sleigh ride and an open air picnic. The closing address was given by Professor Gyula Maksa. His invitation to hold the Twelfth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities in February 2012 in Hungary was gratefully accepted.

Summaries of the talks in alphabetic order of the authors follow in section 1, problems and remarks in section 2, and the list of participants in the final section.

1. Abstracts of talks

Roman Badora: *Stability of a Pexider-type functional equation* (Joint work with Barbara Przebieracz and Peter Volkmann)

We study the stability of the following functional equation

$$f(x\cdot y)=g(x)h(y)+k(y),$$

where f,g,h and k are complex-valued functions defined on a semigroup with unit.

Szabolcs Baják: *On the equality problem in a general class of means* (Joint work with Zsolt Páles)

For a given Borel probability measure μ on $\mathbb{R}$, symmetric with respect to $\frac{1}{2}$ and parameters $r,s\in\mathbb{R}$, $r\neq s$, the two variable symmetric function $M_{r,s,\mu}$ is defined by

$$M_{r,s,\mu}(x,y)=\left(\frac{\int\limits_{-\infty}^{+\infty}\left(x^t y^{1-t}\right)^r d\mu(t)}{\int\limits_{-\infty}^{+\infty}\left(x^t y^{1-t}\right)^s d\mu(t)}\right)^{\frac{1}{r-s}}.$$

With the help of the computer algebra system Maple V Release 9, we investigate the equality problem for this general class of means, i.e., we consider the equation $M_{a,b,\mu}(x,y)=M_{c,d,\mu}(x,y)$.

Karol Baron: *On involutions and Hamel bases*

Answering a (private) question of Bartłomiej Ulewicz we provide the form of all continuous involutions $f\colon\mathbb{R}\to\mathbb{R}$ such that $f(H)\cap H=\emptyset$ for any Hamel basis H of $\mathbb{R}$.

Mihály Bessenyei: *Functional equations and finite group of substitutions I* (Joint work with Csaba G. Kézi)

Motivated by a solving method of certain functional equations known from competition exercises, we investigate the functional equation

$$F(f\circ g_1,\dots,f\circ g_n,\mathrm{id})=0,$$

where F and $g_1,\dots,g_n$ are given (with appropriate range and domain) such that the latter ones form a group under the operation of composition and id denotes the identity of $\mathbb{R}$.

The aim of the talk is to describe the group involved considering the extra regularity properties that should be assumed in order to prove the main result.

Zoltán Boros: *Note on non-negative quadratic functions*

A function $f:\mathbb{R}\to\mathbb{R}$ is called *quadratic* if it fulfils the functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y)$$

for every $x,y\in\mathbb{R}$. Assuming that $f:\mathbb{R}\to\mathbb{R}$ is quadratic and f is non-negative on a Lebesgue measurable set with positive Lebesgue measure, we establish that f has to be non-negative everywhere. We note that this property does not imply the measurability of f.

Zoltán Daróczy: *On a functional equation with a symmetric component* (Joint work with Judita Dascal)

Let $I\subset\mathbb{R}$ be a nonvoid open interval and $A,B,C\in\mathbb{R}$ with

$$ABC\neq 0,\quad (A+C)(B+C)\neq 0,\quad A+B\neq 0,\quad (A+C)+(B+C)\neq 0.$$

Furthermore, let $M:I^2\to I$ be a symmetric mean. In this talk we investigate the functional equation

$$f\left(M(x,y)\right)\left[Ag(y)-Bg(x)\right]=(A+C)f(x)g(y)-(B+C)f(y)g(x)\quad (x,y\in I),$$

where $f,g:I\to\mathbb{R}_+$ are unknown functions.

Włodzimierz Fechner: *Four inequalities of Volkmann type*

We deal with the following four functional inequalities:

$$\max\{f(x+y),f(x-y)\}\leq f(x)+f(y)\quad \text{(for each } x,y)$$
$$\max\{f(x+y),f(x-y)\}\geq f(x)+f(y)\quad \text{(for each } x,y)$$

$$\min\{f(x+y), f(x-y)\} \geq |f(x) - f(y)| \quad \text{(for each } x, y)$$
$$\min\{f(x+y), f(x-y)\} \leq |f(x) - f(y)| \quad \text{(for each } x, y)$$

where the unknown mapping f is defined on an arbitrary Abelian group. Our study is motivated by results of Alice Chaljub-Simon and Peter Volkmann [1] and by several later results concerning the following two functional equations:

$$\max\{f(x+y), f(x-y)\} = f(x) + f(y) \quad \text{(for each } x, y)$$
$$\min\{f(x+y), f(x-y)\} = |f(x) - f(y)| \quad \text{(for each } x, y).$$

Reference

[1] Chaljub-Simon A., Volkmann P., *Caractérisation du module d'une fonction additive a l'aide d'une équation fonctionnelle*, Aequationes Math. **47** (1994), no. 1, 60–68.

Roman Ger: *On vector Hermite–Hadamard differences controlled by their scalar counterparts* (Joint work with Josip Pečarić)

We present a new, in a sense direct, proof that the system of two functional inequalities

$$\left\| F\left(\frac{x+y}{2}\right) - \frac{1}{y-x}\int_x^y F(t)\,dt \right\| \leq \frac{1}{y-x}\int_x^y f(t)\,dt - f\left(\frac{x+y}{2}\right)$$

and

$$\left\| \frac{F(x)+F(y)}{2} - \frac{1}{y-x}\int_x^y F(t)\,dt \right\| \leq \frac{f(x)+f(y)}{2} - \frac{1}{y-x}\int_x^y f(t)\,dt$$

is satisfied for functions F and f mapping an open interval I of the real line $\mathbb{R}$ into a normed linear space and into $\mathbb{R}$, respectively, if and only if F yields a delta-convex mapping with a control function f.

A similar result is obtained for delta-convexity of higher orders with detailed proofs given in the case of delta-convexity of the second order, i.e. when the functional inequality

$$\left\| 3F\left(\frac{x+2y}{3}\right) + F(x) - 3F\left(\frac{2x+y}{3}\right) - F(y) \right\|$$
$$\leq 3f\left(\frac{2x+y}{3}\right) + f(y) - 3f\left(\frac{x+2y}{3}\right) - f(x)$$

holds true provided that $x, y \in I$, $x \leq y$.

Attila Gilányi: *Stability of linear functional equations and completeness of normed spaces* (Joint work with Mohammad Sal Moslehian)

According to Jens Schwaiger's result [4], if E is a normed space then the stability of Cauchy's functional equation considered for functions $f\colon \mathbb{Z} \to E$ implies the completeness of E. During the Conference on Inequalities and Applications '07 in 2007, Mohammad Sal Moslehian formulated the problem whether a similar property is valid for the square norm equation (cf. [2]). This question was answered in the affirmative by Roman Ger during the same meeting and, in a more general setting, by Abbas Najati in [3]. In the paper [1], an analogous theorem was presented for monomial functional equations on the set of positive integers. In the present talk, we prove a generalization of this result for linear functional equations.

References

[1] Ger R., Gilányi A., Volkmann P., *1. Remark (Completeness of normed spaces as a consequence of the stability of some functional equations)*, Report of Meeting, Ann. Math. Sil. **23** (2009), 112–113.
[2] Moslehian M.S., *3. Problem*, Inequalities and Applications (Eds. C. Bandle, et al.), Birkhäuser Verlag, Basel, 2009.
[3] Najati A., *On the completeness of normed spaces*, Applied Math. Letters **23** (2010), 880–882.
[4] Schwaiger J., *12. Remark*, Report of Meeting, Aequationes Math. **35** (1988), 120–121.

Eszter Gselmann: *Approximate f-homomorphisms and f-derivations*

In my talk I would like to present some results on f-homomorphisms as well as approximate f-derivations.

Let $f \in \mathbb{Z}\langle x_1, x_2, \ldots\rangle$ be an arbitrarily fixed multilinear polynomial of degree $m \geq 2$. Furthermore, let $\mathcal{B}$ and $\mathcal{Q}$ be rings and let $\mathcal{S}$ be an f-closed additive subgroup of $\mathcal{B}$ (meaning that $f(x_1, \ldots, x_m) \in \mathcal{S}$ for all $(x_1, \ldots, x_m) \in \mathcal{S}^m$). An additive map $\alpha\colon \mathcal{S} \to \mathcal{Q}$ is called an *f-homomorphism*, if

$$f(x_1, \ldots, x_m)^\alpha = f\left(x_1^\alpha, \ldots, x_m^\alpha\right)$$

holds for all $x_1, \ldots, x_m \in \mathcal{S}$.

Analogously, we define an *f-derivation* to be an additive map $\delta\colon \mathcal{A} \to \mathcal{Q}$ such that

$$f(x_1, \ldots, x_m)^\delta = \sum_{i=1}^{n} f\left(x_1, \ldots, x_{i-1}, x_i^\delta, x_{i+1}, \ldots, x_m\right)$$

for all $x_1, \ldots, x_m \in \mathcal{A}$, where $\mathcal{A}$ is an f-closed additive subgroup of $\mathcal{Q}$.

Let us note that this two notions involve several well–known morphisms e.g. homomorphisms, anti-, Lie-, Jordan-homomorphisms, and the same kind of derivations.

The main result of the talk is to investigate the following problem: Let $\mathcal{B}$ and $\mathcal{Q}$ be topological rings and assume that the additive map $\alpha\colon \mathcal{S} \to \mathcal{B}$ is such that the mapping

$$(x_1, \dots, x_m) \longmapsto f(x_1, \dots, x_m)^\alpha - f\left(x_1^\alpha, \dots, x_m^\alpha\right)$$

satisfies some mild regularity assumption (e.g., boundedness, continuity at a point, measurability). We will investigate that in this case how far is the function α from being an f-homomorphism.

Furthermore, the analogous problem for f-derivations will also be considered.

Csaba G. Kézi: *Functional equations and finite group of substitutions II* (Joint work with M. Bessenyei)

Motivated by a solving method of certain functional equations known from competition exercises, we investigate the functional equation

$$F(f \circ g_1, \dots, f \circ g_n, \mathrm{id}) = 0,$$

where F and $g_1, \dots, g_n$ are given (with appropriate range and domain) such that the latter ones form a group under the operation of composition and id denotes the identity of $\mathbb{R}$.

The aim of the talk is to give a local existence theorem for the solution f under some regularity properties on the known function. In the proof, the Inverse Function Theorem and the Global Existence and Uniqueness Theorem play the key role.

Tomasz Kochanek: *Stability of vector measures and twisted sums of Banach spaces*

We deal with a Hyers–Ulam stability-type problem for vector measures. Let us say that a real Banach space X has the *SVM* (*stability of vector measures*) *property* if and only if there exists an absolute constant $v(X) < +\infty$ (depending only on X) such that given any $\varepsilon > 0$, a set Ω and an algebra $\mathcal{A} \subset 2^\Omega$, and any mapping $\nu\colon \mathcal{A} \to X$ satisfying

$$\|\nu(A \cup B) - \nu(A) - \nu(B)\| \le \varepsilon \quad \text{for } A, B \in \mathcal{A}, A \cap B = \emptyset,$$

there exists a vector measure $\mu\colon \mathcal{A} \to X$ satisfying

$$\|\nu(A) - \mu(A)\| \le v(X)\varepsilon \quad \text{for } A \in \mathcal{A}.$$

Our motivation stems from two sources. The first of these is the Kalton–Roberts theorem ([4, Theorem 4.1]) on nearly additive set functions, which asserts – in our terminology – that the one-dimensional space $\mathbb{R}$ (and then, of course, each finite-dimensional space) has the SVM property. This result was used to establish that the spaces c_0 and ℓ_∞ are the so-called $\mathcal{K}$-spaces, i.e. the only extensions (in the class of locally bounded F-spaces) of $\mathbb{R}$ either by c_0 and ℓ_∞ are the trivial ones: $\mathbb{R} \oplus c_0$ and $\mathbb{R} \oplus \ell_\infty$, respectively. Our research attempt to give an appropriate generalization of the Kalton–Roberts theorem in the setting of vector measures. Another motivation stems from the fundamental papers [2] and [3] establishing a deep general link between splitting and extension properties of Banach spaces, and some stability effects for quasi-linear and zero-linear maps (see also [1]). Our result sheds light on similar connections between stability problem for vector measures and existence of non-trivial twisted sums of Banach spaces. For instance, we prove that a Banach space X, which is isometric to some dual space, has the SVM property if and only if every Banach space which is a twisted sum of ℓ_1 and X^* has to be trivial, i.e. isomorphic to $\ell_1 \oplus X^*$.

References

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[3] Kalton N.J., Peck N.T., *Twisted sums of sequence spaces and the three space problem*, Trans. Amer. Math. Soc. **255** (1979), 1–30.
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Barbara Koclęga-Kulpa: *On a functional equation connected to Hermite quadrature rule* (Joint work with Tomasz Szostok)

Hermite quadrature rule is used in numerical integration for approximating the definite integral in the following way

$$\int_x^y f(t)dt \approx \frac{y-x}{2}[f(x)+f(y)] + \frac{(y-x)^2}{12}[f'(x)-f'(y)].$$

Motivated by this formula we study the functional equation

$$F(y)-F(x) = \frac{y-x}{2}[f(x)+f(y)] + \frac{(y-x)^2}{12}[g(y)-g(x)].$$

It is shown that if f, g, F satisfy this equation then: g must be a polynomial of degree at most 2, f is a polynomial of degree at most 3 and F is a polynomial of degree at most 4. Moreover, $F' = 2f$ and there exists a constant c such that $f' = g + c$.

Judit Makó: *Approximate Hermite–Hadamard inequality* (Joint work with Zsolt Páles)

In this talk, we will investigate the connection between the following functional inequalities

$$f\Big(\frac{x+y}{2}\Big) \le \frac{f(x)+f(y)}{2} + \alpha_J(x-y) \quad x,y \in D,$$

$$\int_0^1 f\big(tx+(1-t)y\big)\rho(t)dt \le \lambda f(x) + (1-\lambda)f(y) + \alpha_H(x-y), \quad x,y \in D,$$

where D is a convex subset of a linear space, $f\colon D \to \mathbb{R}$, $\alpha_H, \alpha_J\colon (D-D) \to \mathbb{R}$ are even functions, $\lambda \in [0,1]$, and $\rho\colon [0,1] \to \mathbb{R}_+$ is an integrable nonnegative function with $\int_0^1 \rho(t)dt = 1$.

Gyula Maksa: *An application of Wigner's theorem in solving some functional equations* (Joint work with Zsolt Páles)

In this talk, we present the general solution of the functional equation

$$\big\{\|f(x)+f(y)\|, \|f(x)-f(y)\|\big\} = \big\{\|x+y\|, \|x-y\|\big\}, \quad x,y \in X,$$

where $f\colon X \to Y$ and X, Y are inner product spaces. Related equations, such as

$$\begin{aligned} \|f(x)+f(y)\| + \|f(x)-f(y)\| &= \|x+y\| + \|x-y\|, \quad x,y \in X, \\ \|f(x)+f(y)\|\|f(x)-f(y)\| &= \|x+y\|\|x-y\|, \quad x,y \in X, \\ |\Re\langle f(x), f(y)\rangle| &= |\Re\langle x,y\rangle|, \quad x,y \in X, \end{aligned}$$

will also be considered. In the investigations, the main tool is a real version of Wigner's unitary-antiunitary theorem.

Janusz Matkowski: *Generalized subadditive functions - monotonicity and periodicity*

The continuity, monotonicity, and periodicity of functions satisfying the inequality

$$f(\alpha x + y) \le a f(x) + f(y),$$

where $\alpha, a > 0$ are arbitrarily fixed, will be considered.

Fruzsina Mészáros: *Density function solutions of the Olkin–Baker equation* (Joint work with Károly Lajkó)

Since it remained as an open problem in [2], we will give the so-called density function solutions of the Olkin-Baker equation

$$f(x)\,g(y) = p(x+y)\,q\left(\frac{x}{y}\right) \qquad \text{for a.a. } (x,y) \in \mathbb{R}_+^2.$$

To do so, we will use the previous results for a general multiplicative type equation satisfied almost everywhere (see [1]).

References

[1] Járai A., Lajkó K., Mészáros F., *On measurable functions satisfying multiplicative type functional equations almost everywhere* Inequalities and Applications '10, International Series of Numerical Mathematics, Birkhauser Verlag, submitted.

[2] Mészáros F., *A functional equation and its application to the characterization of gamma distributions*, Aequationes Math. **79** (2010), 53–59.

Lajos Molnár: *Transformations on self-adjoint operators preserving a measure of commutativity* (Joint work with W. Timmermann)

In this talk we describe the structure of all bijective nonlinear maps on the space of all bounded self-adjoint operators acting on a complex separable Hilbert space of dimension at least 3 which preserve a sort of measure of commutativity, namely, the norm of the commutator of operators. It turns out that those transformations are closely related to Jordan automorphisms.

Janusz Morawiec: *Refinement equations and distributional fixed points* (Joint work with Rafał Kapica)

Assume that $(\Omega, \mathcal{A}, P)$ is a complete probability space and $K, L\colon \Omega \to \mathbb{R}$ are random variables. We investigate the connections between $\mathbb{L}^1$-solutions of the refinement equation

$$f(x) = \int_\Omega |L(\omega)| f(K(\omega)x - L(\omega))\,dP(\omega)$$

and distributional fixed points of a special random affine map.

Agata Nowak: *Chini's equations* (Joint work with Maciej Sablik)

In 1839 De Morgan gave a mathematical justification of Gompertz's law of mortality through a composite functional equation

$$F(x+y) + F(x+z) = F(x + u(y,z)), \quad x, y, z \in \mathbb{R}. \tag{1}$$

A slightly more general version of this equation i.e.

$$F(x+y)+F(x+z)=G(x+u(y,z)),\quad x,y,z\in\mathbb{R}, \tag{2}$$

as well as its multiplicative version, i.e.

$$f(x+y)f(x+z)=f(x+u(y,z)),\quad x,y,z\in\mathbb{R},$$

was studied in 1905 by M. Chini in connection with actuarial mathematics. Both of them solved equation (1) in the class of differentiable functions on the real line. In 2001 T. Riedel, M. Sablik and P. K. Sahoo found solutions of (2) under less restrictive assumptions.

We discuss a motivation of the equation (1) and a possibility of simplifying the equation

$$f(x+y)f(x+z)=\psi(x+u(y,z)),\quad x,y,z\in\mathbb{R},$$

to the equation (2).

Andrzej Olbryś: *On delta t-convex and Wright-convex mappings*

Let $(X,||\cdot||)$ and $Y,||\cdot||)$ be two real Banach spaces, and let D be a nonempty open and convex subset of X. Following L. Veselý and L. Zajiček [3] also R. Ger [1, 2] we introduce the following definitions :

A map $F\colon D\to Y$ is termed *delta t-convex* (where $t\in(0,1)$ is fixed number) which a control function $f\colon D\to\mathbb{R}$, if

$$\bigwedge_{x,y\in D}\|tF(x)+(1-t)F(y)-F(tx+(1-t)y)\|\le tf(x)+(1-t)f(y)-f(tx+(1-t)y).$$

If the above inequality is satisfied for all $t\in[0,1]$ then we say that F is a *delta convex* mapping [3].

A map $F\colon D\to Y$ is termed *delta Wright-convex* which a control function $f\colon D\to\mathbb{R}$, if for all $x,y\in D$ and $t\in[0,1]$ the following inequality holds

$$\begin{aligned}&\|F(x)+F(y)-F(tx+(1-t)y)-F((1-t)x+ty)\|\\&\qquad\le f(x)+f(y)-f(tx+(1-t)y)-f((1-t)x+ty).\end{aligned}$$

In our talk we present some results concerning defined mappings, in particular we present a support theorem for delta t-convex maps. As a consequence of this theorem we obtain a characterization of delta Wright-convex mappings in the following way: F is a delta Wright-convex map, if and only if, it is a sum of additive and delta convex mapping.

References

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[3] Veselý L., Zajiček L., *Delta-convex mappings between Banach spaces and applications*, Dissertationes Math. 289, Polish Scientific Publishers, Warszawa, 1989.

Zsolt Páles: *A maximum theorem for generalized convex functions*

Given two convex functions $f, g\colon D \to \mathbb{R}$ with property

$$0 \le \max(f(x), g(x)), \quad x \in D,$$

it is well known that there exists $\lambda \in [0,1]$ such that

$$0 \le \lambda f(x) + (1-\lambda)g(x), \quad x \in D.$$

An analogous result is known for subadditive functions f and g.

The aim is to extended this theorem to a class of functions which generalizes both convexity and subadditivity.

Barbara Przebieracz: *Stability of the translation equation*

We show the stability of the translation equation

$$F(t, F(s,x)) = F(s+t, x), \quad s,t \in \mathbb{R}, x \in I,$$

where $I \subset \mathbb{R}$ is a real interval, in the class of continuous functions.

Maciej Sablik: *Aggregation of transitive relations* (Joint work with Jolanta Sobera)

Let T be a t-norm, let $R_1, \dots, R_n\colon X \times X \to [0,1]$ be fuzzy relations (i.e. functions with values in $[0,1]$), finally, let $F\colon [0,1]^n \to \mathbb{R}$ be a mean, i.e. nondecreasing function with $F(0,\dots,0) = 0$ and $F(1,\dots,1) = 1$. We say that a fuzzy relation $R\colon X \times X \to X$ is sup-T transitive if for every $x, z \in X$ we have $\sup\{T(R(x,y), R(y,z)) : y \in X\} \le R(x,z)$. We ask about transitivity of relations defined by $R(x,y) = F(R_1(x,y), \dots, R_n(x,y))$ for different functions F and t-norms T.

László Székelyhidi: *A Moment Problem on Hypergroups I* (Joint work with László Vajday)

In 1894 Thomas Jan Stieltjes published an extremely influential paper: *Recherches sur les fractions continues,* Ann. Fac. Sci. Toulouse, **8**, 1122; **9**, 547. He introduced what is now known as the Stieltjes integral with respect to an increasing function. This integral was used to solve the problem which we call now the moment problem. Since then different types of moment problems have been studied and solved by different authors. In this talk we formulate a moment problem on hypergroups.

PATRÍCIA SZOKOL: *Kolmogorov–Smirnov isometries of the space of generalized distribution functions* (Joint work with Lajos Molnár)

In the paper *Isometries of the space of distribution functions with respect to the Kolmogorov-Smirnov metric* (J. Math. Anal. Appl. 348 (2008), 494–498.) Gregor Dolinar and Lajos Molnár have described the general forms of surjective isometries of the space of all probability distribution functions on $\mathbb{R}$ with respect to the Kolmogorov–Smirnov metric.

In this talk we consider the space of so-called generalized probability distribution functions (i.e. functions from $\mathbb{R}$ to $[0,1]$ which are monotone increasing and continuous from the right without restrictions concerning the limits at $\pm\infty$). We show that the structure of the surjective isometries of that space with respect to the Kolmogorov–Smirnov metric is similar to that of the surjective isometries of the space of classical probability distribution functions.

TOMASZ SZOSTOK: *Some functional equations characterizing polynomials* (Joint work with Maciej Sablik)

Recently at the Seminar on Functional Equations W. Fechner presented results of a joint work with E. Gselmann concerning solutions of the equation

$$F(x+y) - F(x) - F(y) = xf(y) + yf(x)$$

and of the related inequality.

In this talk we show how the solutions of this equation may be obtained with use of a lemma contained in [1] and we study more general equations:

$$F(x+y) - F(x) - F(y) = x\sum_{n=1}^{k} \alpha_n f(a_n x + b_n y) + y\beta f(x)$$

and

$$\sum_{n=1}^{l} \gamma_n F(A_n x + B_n y) = x\sum_{n=1}^{k} \alpha_n f(a_n x + b_n y) + y\beta f(x).$$

Reference

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László Vajday: *A Moment Problem on Hypergroups II* (Joint work with László Székelyhidi)

In this talk we present some new results on polynomial and Sturm–Liouville hypergroups. We focus on the uniqueness of the moment problem, which is based on a special orthogonality. More precisely, if a compactly supported measure is considered on the base set of the actual hypergroup, what properties have to be formulated that the integrals of a given sequence of moment functions with respect to the measure mentioned before will be equal to zero. In a different approach the problem is the following: when will a compactly supported measure be orthogonal to all functions in a given moment sequence? The problem is formulated on hypergroups mentioned above and in this talk we give the solution of the uniqueness problem for polynomial hypergroups in a single variable and for Sturm–Liouville hypergroups.

Peter Volkmann: *Which groups are Tabor groupoids* (Joint work with Roman Badora and Barbara Przebieracz)

Let S be a groupoid. For $x \in S$ and $k = 0, 1, 2, \dots$ the powers x^{2^k} are recursively defined by $x^{2^0} = x^1 = x, x^{2^{k+1}} = x^{2^k} x^{2^k}$. Józef Tabor (1984/85) pointed out the usefulness of the following condition for stability investigations:

(T) If $x, y \in S$, then there is $k \geq 1$ such that $(xy)^{2^k} = x^{2^k} y^{2^k}$.

Groupoids satisfying (T) are called Tabor groupoids in [1]; there and also in [2] stability results for Tabor groupoids are given. Here we give examples of groups S where (T) holds, and of groups where (T) does not hold.

References

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2. Problems and Remarks

1. Remark. *(Stability of the functional equation $f(xy) = f(yx)$ on groups.)*

Theorem. *Let G be a group, E a normed space, and suppose $f : G \to E$ to satisfy*

$$\|f(xy) - f(yx)\| \leq \varepsilon \quad (x, y \in G). \tag{1}$$

Then there exists $g : G \to E$ such that

$$g(xy) = g(yx), \ \ \|g(x) - f(x)\| \leq \varepsilon \quad (x, y \in G). \tag{2}$$

Proof. (1) can be written as

$$\|f(z) - f(yzy^{-1})\| \leq \varepsilon \quad (y, z \in G). \tag{3}$$

In G we get an equivalence relation, when defining $x \sim z$ by the existence of $y \in G$ such that $x = yzy^{-1}$. Then (3) means

$$\|f(z) - f(x)\| \leq \varepsilon \quad (x, z \in G, \ x \sim z).$$

Let $\mathcal{A}$ be the set of equivalence classes, in every $A \in \mathcal{A}$ choose $z_A \in A$, and define $g : G \to E$ by $g(x) = f(z_A)$ $(x \in A \in \mathcal{A})$. Then g fulfils (2). $\square$

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