Report of Meeting

The Fourth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities February 4–7, 2004 Mátraháza, Hungary

The Fourth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities was held in Mátraháza, Hungary from February 4 to February 7, 2004, at the Resort House of the Hungarian Academy of Sciences. It was organized by the Institute of Mathematics of the University of Debrecen, with the financial support of the Hungarian Scientific Research Fund OTKA T-043080. The organizers were ably assisted by Ms. Borbála Fazekas and Ms. Ágota Orosz.

24 participants came from the University of Debrecen (Hungary) and the Silesian University of Katowice (Poland) at 12 from each of both cities.

Professor Zsolt Páles opened the Seminar and welcomed the participants to Mátraháza.

The scientific talks presented at the Seminar focused on the following topics: equations in a single and several variables, iteration theory, equations on algebraic structures, geometric preservers, Hyers–Ulam stability, functional inequalities and mean values, generalized convexity. Interesting discussions were generated by the talks.

There were three very profitable Problem Sessions.

The social program included a guided tour in the town Gyöngyös involving a visit to the Mátra Museum, wine tasting at a cellar in Abasár, and a festive dinner.

The closing address was given by Professor Roman Ger. His invitation to the Fifth Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities in February 2005 in Poland was gratefully accepted.

Summaries of the talks in alphabetic order of the authors follow in section 1, problems and remarks in approximate chronological order in section 2, and the list of participants in the final section.

1. Abstracts of talks

Roman Badora: *Approximate isometric operators*
(Joint work with Jacek Chmieliński)

In this talk we present some remarks on mappings preserving the inner product approximately. Among others, we prove the following

Theorem. *Let E, F be Hilbert spaces and let $\varepsilon \geq 0$. Then $f\colon E \longrightarrow F$ satisfies*

$$|\langle f(x)|f(y)\rangle - \langle x|y\rangle| \leq \varepsilon, \qquad x, y \in E$$

if and only if there exists a closed linear subspace H of F such that

$$f(x) = P_{H^\perp} f(x) + P_H f(x), \qquad x \in E,$$

$$\|P_{H^\perp} f(x)\| \leq \sqrt{\varepsilon}, \qquad x \in E$$

and

$$\langle P_H f(x)|P_H f(y)\rangle = \langle x|y\rangle, \qquad x, y \in E.$$

Here, for a closed linear subspace S of F we define

$$S^\perp := \{w \in F : \ w \perp v \text{ for all } v \in S\}$$

and for $u \in F$ by $P_S u$ we denote the orthogonal projection of u on S.

Lech Bartłomiejczyk: *On the set of derivations with big graph*

In the topological vector space of all derivations from $\mathbb{R}$ to $\mathbb{R}$ we deal with a set of derivations with big graph.

Mihály Bessenyei: *Hermite–Hadamard inequalities for generalized 3-convex functions*
(Joint work with Zsolt Páles)

The aim of the talk is to present Hermite–Hadamard type inequalities for generalized 3-convex functions. A particular result for generalized 4-convex functions is also obtained.

Zoltán Boros: *Strong geometric differentiability and local superstability for a Pexider equation*

Let I denote an open interval in the real line, $p > 1$, and suppose that, for every $x \in I$, $f\colon I \longrightarrow \mathbb{R}$ satisfies an inequality of the form

$$|f(y+u) - f(y) - \phi_x(u)| \leq \varepsilon(x)|u|^p$$

for every y taken from a neighbourhood of x and for every u taken from a neighbourhood of 0. It is derived from this inequality that the finite limit

$$\lim_{\substack{y \to x \\ n \to \infty}} 2^n \left(f(y + 2^{-n}h) - f(y)\right)$$

exists for every $x \in I$ and $h \in \mathbb{R}$. Applying an appropriate decomposition theorem, we obtain that $f = g + \phi$, where g is continuously differentiable and ϕ is the restriction of an additive mapping to the interval I. Substitution into the original stability inequality yields that g' is constant and thus f is affine.

PÉTER CZINDER: *An extension of the Hermite-Hadamard inequality and an application for Gini and Stolarsky means*
(Joint work with Zsolt Páles)

We extend the Hermite–Hadamard inequality

$$f\left(\frac{p+q}{2}\right) \le \frac{1}{q-p}\int_p^q f(x)\,dx \le \frac{f(p)+f(q)}{2}$$

for convex-concave symmetric functions. As consequences some new inequalities for Gini and Stolarsky means are also derived.

ZOLTÁN DARÓCZY: *Quasi-arithmetic means imbedded*

Let $I \subset \mathbb{R}$ be an open interval and let $\mathcal{K}(I)$ be a class of means on I. It is a general problem to determine the means in $\mathcal{K}(I)$ which are quasi-arithmetic. We give a summary on results of this type.

ROMAN GER: *On alternative equations defining ring homomorphisms and derivations*
(Joint work with Ludwig Reich)

Let X be a unitary ring and let Y be a commutative ring with no zero divisors (resp. let X be an integral domain). We study the solutions $f\colon X \longrightarrow Y$ (resp. $f\colon X \longrightarrow X$) of the equation

$$cf(x+y) + df(xy) = cf(x) + cf(y) + df(x)f(y), \qquad x, y \in X,$$

(resp.

$$cf(x+y) + df(xy) = cf(x) + cf(y) + dxf(y) + dyf(x), \qquad x, y \in X).$$

Our main goal is to establish whether or not the solutions have to satisfy the system

$$\begin{cases} f(x+y) = f(x) + f(y) \\ \quad f(xy) = f(x)f(y), \end{cases}$$

defining the ring homomorphisms (resp. the system

$$\begin{cases} f(x+y) = f(x) + f(y) \\ \quad f(xy) = xf(y) + yf(x), \end{cases}$$

defining the derivations in a ring).

The first problem generalizes simultaneously similar questions asked earlier by J. Dhombres ($c = d = 1$) and H. Alzer ($c = 1$, $d = -1$).

ATTILA GILÁNYI: *A general stability theorem for functional equations*
(Joint work with Zsolt Páles)

We investigate the stability of the functional equation

$$F(f(x \diamond_1 y), \dots, f(x \diamond_k y)) = 0 \qquad (x, y \in X),$$

where X is a groupoid with binary operations $\diamond_1, \dots, \diamond_k$, (Y, d) is a complete metric space, $F\colon Y^k \longrightarrow \mathbb{R}^+$ is a given continuous function, and $f\colon X \longrightarrow Y$ is the 'unknown' function.

ATTILA HÁZY: *On approximate t-convexity*

A real valued function f defined on an open convex set D is called $(\varepsilon, \delta, p, t)$-convex if it satisfies

$$f\,(tx + (1-t)y) \le tf(x) + (1-t)f(y) + \delta + \varepsilon|x-y|^p \qquad \text{for } x, y \in D.$$

We prove that if f is locally bounded from above at a point of D and $(\varepsilon, \delta, p, t)$-convex (where $t \le 1/2$) then it satisfies the convexity-type inequality

$$f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda)f(y) + \max\left\{\tfrac{1}{t}, \tfrac{1}{1-t}\right\}\delta + \varepsilon\varphi_{p,t}(\lambda)|x-y|^p$$

for $x, y \in D$, $\lambda \in [0,1]$, where $\varphi_{p,t}$ is the fixed point of the contraction $T_{p,t}$ defined by

$$(T_{p,t}\Phi)(\lambda) = \begin{cases} \min\left\{(1-t)\Phi\left(\tfrac{\lambda}{1-t}\right) + \left(\tfrac{\lambda}{1-t}\right)^p; t\Phi\left(\tfrac{\lambda}{t}\right) + \left(\tfrac{\lambda}{t}\right)^p\right\}, & 0 \le \lambda \le t, \\ \min\left\{(1-t)\Phi\left(\tfrac{\lambda}{1-t}\right) + \left(\tfrac{\lambda}{1-t}\right)^p; (1-t)\Phi\left(\tfrac{1-\lambda}{1-t}\right) + \left(\tfrac{1-\lambda}{1-t}\right)^p\right\}, & t < \lambda < 1-t, \\ \min\left\{t\Phi\left(\tfrac{1-\lambda}{t}\right) + \left(\tfrac{1-\lambda}{t}\right)^p; (1-t)\Phi\left(\tfrac{1-\lambda}{1-t}\right) + \left(\tfrac{1-\lambda}{1-t}\right)^p\right\}, & 1-t \le \lambda \le 1. \end{cases}$$

If $0 < t \le 1/2$ and $0 < p < 1$, then $\varphi_{p,t}$ satisfies the inequality

$$\frac{1}{(t(1-t))^p}\phi_p(\lambda) \le \varphi_{p,t}(\lambda) \le \max\left\{\frac{1}{t^p - t}; \frac{1}{(1/2 - t/2)^p - (1-t)^{1-p}(1/2-t)^p}\right\}\phi_p(\lambda)$$

for all $\lambda \in [0,1]$, where function ϕ_p is defined by

$$\phi_p(\lambda) := (\lambda(1-\lambda))^p.$$

Therefore, if f is locally bounded from above at a point of D and $(\varepsilon, \delta, p, t)$-convex (where $0 \le p < 1$ and $t \le 1/2$), then it satisfies the convexity-type inequality

$$\begin{aligned} f\,(\lambda x + (1-\lambda)y) &\le \lambda f(x) + (1-\lambda)f(y) + (1/t)\delta \\ &\quad + \varepsilon\max\left\{\frac{1}{t^p - t}; \frac{1}{(1-t)^p((1/2)^p - (1-t)^{1-2p}(1/2-t)^p)}\right\}(\lambda(1-\lambda))^p|x-y|^p \end{aligned}$$

for all $x, y \in D$ and $\lambda \in [0,1]$.

In the case $p = 1$, $t = 1/2$ analogous results were obtained in [2]. The case $\varepsilon = 0$ was investigated by Páles in [4], the case $\varepsilon = 0$ and $t = 1/2$ by Nikodem and Ng in [3], the specialization $\varepsilon = \delta = 0$ yields the theorem of Bernstein and Doetsch [1].

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[1] F. Bernstein, G. Doetsch, *Zur Theorie der konvexen Funktionen*, Math. Annalen **76** (1915), 514–526.

[2] A. Házy, Zs. Páles, *Approximately midconvex functions*, Bull. London Math. Soc. (accepted).

[3] C.T. Ng, K. Nikodem, *On approximately convex functions*, Proc. Amer. Math. Soc. **118** (1993), No. 1, 103–108.

[4] Zs. Páles, *Bernstein–Doetsch-type results for general functional inequalities*, Rocznik Nauk.-Dydakt. Prace Mat. **17** (2000), 197–206 (Dedicated to Professor Zenon Moszner on his 70th birthday).

ZOLTÁN KAISER: *Estimates to the stability of the Cauchy equation*
(Joint work with Attila Gilányi and Zsolt Páles)

Suppose we have a function f mapping a groupoid $(S, \diamond)$ into a metric groupoid $(T, *, d)$ and satisfying the stability inequality

$$d\big(f(x \diamond y), f(x) * f(y)\big) \leq \varepsilon(x, y) \qquad (x \in S).$$

We approximate f by a solution g of the equation

$$g(x \diamond y) = g(x) * g(y) \qquad (x \in S)$$

and obtain estimates for the pointwise distance of the approximating function g and the original function f. As application, we deduce some stability theorems for l-power-symmetric groupoids.

BARBARA KOCLĘGA-KULPA: *On a generalization of the Cauchy equation*

We consider the following generalization of the Cauchy equation

$$p(f(x+y)) = p(f(x) + f(y)).$$

Under some assumptions imposed on function p and f we derive the additivity of f. In particular, we obtain the equivalence of the equation of the form

$$f(x+y)^2 + \ldots + f(x+y)^{2n} = [f(x) + f(y)]^2 + \ldots + [f(x) + f(y)]^{2n}$$

to the Cauchy functional equation.

ZYGFRYD KOMINEK: *A few remarks on almost C-polynomial functions*

We give some sufficient conditions to a function transforming a commutative semigroup to a commutative group to be a polynomial function. Some stability results are also given.

Károly Lajkó: *A functional equation related to the characterization of beta distributions*

Recently J. Wesołowski [1] studied a characterization of beta distributions, which leads to the functional equation

$$f_U(u)f_V(v) = \frac{v}{1-uv} f_X\left(\frac{1-v}{1-uv}\right) f_Y(1-uv) \qquad (u,v \in]0,1[), \tag{1}$$

where $f_U, f_V, f_X, f_Y\colon]0,1[\longrightarrow \mathbb{R}$ are the probability density functions of the random variables $U, V, X = \frac{1-V}{1-UV}$, and $Y = 1-UV$, respectively. He solved (1) under the assumption the density functions are strictly positive and locally integrable on $]0,1[$. But Wesołowski noticed that measurability is a more natural property for density functions. Using a well-known method of A. Járai, we present here the general measurable solution of equation (1).

Reference

[1] J. Wesołowski, *On a functional equation related to an independence property for beta distributions*, Aequationes Math. **60** (2003), 156–163.

Grażyna Łydzińska: *Collapsing and expanding iteration semigroups of set-valued functions*

We present some conditions under which a certain family of set-valued functions, naturally occuring in iteration theory, fulfils one of the following conditions

$$F(s+t,x) \subset F(t,F(s,x)), \tag{C}$$
$$F(t,F(s,x)) \subset F(s+t,x) \tag{E}$$

for every $x \in X$, $s,t \in (0,\infty)$ (where X is an arbitrary set). Moreover, we compare the above conditions and answer the question, whether either (C) or (E) implies that F is an iteration semigroup:

$$F(t,F(s,x)) = F(s+t,x)$$

for every $x \in X$, $s,t \in (0,\infty)$.

Gyula Maksa: *Equations of associative type*

In this talk we present the continuous solutions of some functional equations of associative type that are strictly monotonic in each variable. Neither solvability nor differentiability conditions are assumed.

Janusz Matkowski: *On some relations between M-convexity (M-affinity) and N-convexity*

Let M and N be some means in an interval I. The question when, for every continuous function $f\colon I \longrightarrow I$, M-convexity (M-affinity) of f implies its N-convexity is considered.

ZSOLT PÁLES: *Representation of the arithmetic mean as a generalized quasiarithmetic mean with weight function*

Given two continuous functions $f, g\colon I \longrightarrow \mathbb{R}$ such that g is positive and f/g is strictly monotone and a Borel probability measure μ, the generalized quasiarithmetic mean with weight function $M_{f,g;\mu}$ is defined by

$$M_{f,g;\mu}(x,y) = \left(\frac{f}{g}\right)^{-1}\left(\frac{\int_0^1 f(tx+(1-t)y)\,d\mu(t)}{\int_0^1 g(tx+(1-t)y)\,d\mu(t)}\right) \qquad (x,y \in I).$$

Observe that with $\mu = \frac{\delta_0+\delta_1}{2}$ one obtains a quasiarithmetic mean with weight function. If μ is the Lebesgue measure then the so called Cauchy means are obtained.

Our aim is to describe all triplets (f, g, μ) such that

$$M_{f,g;\mu}(x,y) = \frac{x+y}{2} \qquad (x,y \in I).$$

IWONA PAWLIKOWSKA: *Stability of Flett's points of n-th order*

In [1] M. Das, T. Riedel and P.K. Sahoo dealt with Hyers–Ulam stability of Flett's points of differentiable function $f\colon [a,b] \longrightarrow \mathbb{R}$ i.e. intermediate points $\eta \in (a,b)$ such that

$$f(\eta) - f(a) = f'(\eta)(\eta - a).$$

They used the result of S.M. Hyers and D.H. Ulam [2] connected with stability of Rolle's points. We study Flett's points of n-th order for which the following formula holds

$$\begin{aligned} f(\eta) - f(a) = {} & \sum_{k=1}^{n}(-1)^{k-1}\frac{1}{k!}f^{(k)}(\eta)(\eta-a)^k \\ & + (-1)^n\frac{1}{(n+1)!}\frac{f^{(n)}(b)-f^{(n)}(a)}{b-a}(\eta-a)^{n+1} \end{aligned}$$

and we show their stability.

REFERENCES

[1] M. Das, T. Riedel, P.K. Sahoo, *Hyers–Ulam Stability of Flett's points*, Applied Math. Letters **16** (2003), 269–271.

[2] D.H. Hyers, S.M. Ulam, *On the stability of differential expressions*, Math. Magazine **28** (1954), 59–64.

MACIEJ SABLIK: *Aggregating vector valued allocation*

Our aim is to generalize the result from [1] where the authors determined aggregation methods for allocation problems. The authors of [1] considered the situation of assigning numerical values to decision variables while in the present case we admit assignment of vector values, which happens in some decision making procedures. The outcome is, *mutatis mutandis*, analogous to the one obtained in [1].

REFERENCE

[1] J. Aczél, C.T. Ng, C. Wagner, *Aggregation theorems for allocation problems*, SIAM J. Alg. Disc. Meth. 5 (1984), 1–8.

JUSTYNA SIKORSKA: *On mappings preserving equilateral triangles in normed spaces* (Joint work with Tomasz Szostok)

Let $(X, \|\cdot\|)$ and $(Y\|\cdot\|)$ be normed linear spaces, $\dim X, \dim Y \geq 2$. We say that $f\colon X \longrightarrow Y$ *preserves equilateral triangles* if for all triples of points $x, y, z \in X$ with $\|x-y\| = \|y-z\| = \|x-z\|$ we have

$$\|f(x) - f(y)\| = \|f(y) - f(z)\| = \|f(x) - f(z)\|.$$

We show that if X and Y are at least three-dimensional and $f\colon X \longrightarrow Y$ is surjective and preserves equilateral triangles, then it is a similarity transformation (an isometry multiplied by a positive constant).

We prove also some new results in case $X = Y$ is an inner product space with $\dim X = 2$.

LÁSZLÓ SZÉKELYHIDI: *Binomial functional equations on polynomial hypergroups* (Joint work with Ágota Orosz)

The study of generalized moment functions on polynomial hypergroups in a single variable started with some results of Á. Orosz and L. Székelyhidi presented at ISFE 40' in Sandbjerg, Danmark, 2002. At ISFE 41' in Noszvaj, Hungary, 2003, we exhibited the complete description of moment functions on polynomial hypergroups in one variable. This problem is closely related to the study of binomial functional equations. Since the Noszvaj meeting there have been new developments in the case of polynomial hypergroups in several variables which are presented in this talk.

TOMASZ SZOSTOK: *On ω-convex functions*

Let $\emptyset \neq D \subset \mathbb{R}$ be a given subset and $\omega\colon D \longrightarrow \mathbb{R}$ be a function. We say that ω has the *joining points* property if and only if ω is continuous and for every pair $(a,b) \in \mathbb{R}^2$ with $a > 0$ there exists exactly one $x \in D$ such that

$$[x, x+a] \subset D \quad \text{and} \quad \omega(x+a) - \omega(x) = b.$$

In such a case we say shortly that ω is a JP-function. Now fix a JP-function ω and consider some function $f\colon \mathbb{R} \to \mathbb{R}$. The function f is called ω-convex if and only if for all $x, y \in \mathbb{R}$, $x < y$ and for every $z \in (x,y)$ we have

$$f(z) \leq \omega(z+\alpha) + \beta,$$

where $\alpha, \beta \in \mathbb{R}$ are such that $\omega(x+\alpha) + \beta = f(x)$ and $\omega(y+\alpha)+\beta = f(y)$. Such numbers clearly exist and are uniquely determined since ω is a JP-function and therefore the above definition is correct. In the similar way we define also Jensen ω-convexity and our main result states that a continuous function which is Jensen ω-convex has to be ω-convex. We describe also the form of a JP-function and we determine all ω-convex functions in the case $\omega(x) = x^2$.

JANUSZ WALORSKI: *On continuous and homeomorphic solutions of the Schröder equation in Banach spaces*

Let X be a Banach space. We consider the Schröder equation

$$\varphi(f(x)) = A\varphi(x)$$

in which $\varphi: X \longrightarrow X$ is an unknown function, the linear operator $A: X \longrightarrow X$ and the function $f: X \longrightarrow X$ are given. We establish conditions under which there exists a continuous (homeomorphic) solution of the above equation.

2. Problems and Remarks

1. PROBLEM. The logarithmic mean L is defined by

$$L(x,y) := \begin{cases} \frac{x-y}{\ln x - \ln y}, & \text{if } x,y \in (0,\infty),\ x \neq y, \\ x, & \text{if } x = y,\ x \in (0,\infty). \end{cases}$$

Question 1. Does there exist a discontinuous solution $f: (0,\infty) \longrightarrow (0,\infty)$ of the functional equation

$$(1) \qquad f\left(L(x,y)\right) = L\left(f(x), f(y)\right) \qquad (x,y \in (0,\infty))\,?$$

Question 2. Is L-convexity (or L-affinity) a localizable property in the following sense: if, for every $u \in (0,\infty)$, there exists an open interval $U \subset (0,\infty)$ such that $u \in U$ and

$$(2) \qquad f\left(L(x,y)\right) \leq L\left(f(x), f(y)\right) \qquad \text{for every } x,y \in U$$

(respectively, $f(L(x,y)) = L(f(x), f(y))$ for every $x, y \in U$), then f is L-convex (respectively, L-affine)?

ZSOLT PÁLES

2. REMARK AND PROBLEM. According to the axioms of the real number system, the operation $x + y$ for $x, y \in \mathbb{R}$ (the usual addition) is associative. Moreover, if, for instance, $\varphi: \mathbb{R} \longrightarrow \mathbb{R}$ is bijective, then also the operation $*$ defined by

$$(1) \qquad x * y = \varphi^{-1}\left(\varphi(x) + \varphi(y)\right) \qquad (x,y \in \mathbb{R})$$

is associative. According to a celebrated result by János Aczél, if an operation $*: \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ is associative, cancellative, and continuous, then there exists a mapping φ such that $*$ is given by (1).

It was presented in our joint remark with A. Gilányi at the 41st ISFE that if $H: \mathbb{R}^2 \longrightarrow \mathbb{R}$ is homogeneous, i.e., $H(tx, ty) = tH(x,y)$ is fulfilled for every $t, x, y \in \mathbb{R}$, then the operation $\circ: \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ defined by $x \circ y = H(x,y)$ $(x, y \in \mathbb{R})$ is l-power symmetric for every positive integer l. Moreover, if $\varphi: \mathbb{R} \longrightarrow \mathbb{R}$ is bijective, then also the operation $*$ defined by

$$(2) \qquad x * y = \varphi^{-1}\left(H(\varphi(x), \varphi(y))\right) \qquad (x,y \in \mathbb{R})$$

is l-power symmetric. In analogy to Aczél's above mentioned result, it is natural the formulate the following problem: Find reasonable sufficient conditions under which every power symmetric (i.e., l-power symmetric for every $l \in \mathbb{N}$) operation $*\colon \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ can be written in the form (2).

ZSOLT PÁLES

3. PROBLEM. A function $f\colon (0,\infty) \longrightarrow (0,\infty)$ is called convex with respect to the logarithmic mean

$$L(x,y) := \begin{cases} \dfrac{x-y}{\ln x - \ln y}, & \text{if } x,y \in (0,\infty),\ x \neq y, \\ x, & \text{if } x = y,\ x \in (0,\infty) \end{cases}$$

if f fulfills the inequality

$$f\left(L(x,y)\right) \leq L\left(f(x), f(y)\right), \qquad x,y \in (0,\infty). \tag{1}$$

It is known that every decreasing function f fulfilling (1) is convex in the usual sense. The question is whether every continuous function f fulfilling (1) has to be convex in the usual sense.

ZYGFRYD KOMINEK

4. REMARK (to 3. Problem by Z. Kominek). The following partial answer to Kominek's question can be derived: if $f\colon (0,\infty) \longrightarrow (0,\infty)$ is twice continuously differentiable and L-convex, then the mapping

$$x \mapsto \sqrt[3]{f(x^3)} \qquad (x \in (0,\infty))$$

is convex in the usual sense. Moreover, if the logarithmic mean L is replaced by the Stolarsky mean with parameters (p,q), then an analogous result can be obtained, where the exponent 3 is substituted by $\frac{3}{p+q}$.

ZSOLT PÁLES

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